

Radiative Muon Capture Yield in Nuclei

Inaugural Dissertation

zur Erlangung der philosophischen Doktorwürde
vorgelegt der
Philosophischen Fakultät II
der
Universität Zürich

von

Max Döbeli
aus Meisterschwanden AG

begutachtet von Herrn Prof. Dr. Peter Truöl

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ABSTRACT

The present work reports the results of measurements of the photon spectra and the absolute branching ratios of radiative muon capture (RMC) in ^{12}C , ^{16}O , ^{40}Ca , ^{27}Al , ^{nat}Fe , ^{165}Ho , ^{209}Bi , performed in 1984 at the Swiss Institute for Nuclear Research (SIN) at Villigen. These branching ratios may be regarded as a probe of the induced pseudoscalar coupling of the semileptonic weak interaction in the nucleus.

According to the presently existing RMC calculations and nuclear models it is found that g_P deviates from the canonical value of $7g_A$ for medium heavy nuclei and is nearly totally quenched for high values of Z .

These results on the one hand can be interpreted as further confirmation of the predicted renormalization of the nucleon formfactors inside the nucleus, or on the other hand as an indication of the insufficiency of the nuclear model calculations.

The following short chapters contain a concise description of the theoretical background and the experimental set-up as well as a compilation of the results. For those who want to bore themselves with particulars, some larger chapters with the details follow. Last and least follows an appendix for real RMC enthusiasts.

ACKNOWLEDGEMENTS

I am indebted to the following persons and institutions for their contribution to this work:

- Professor Peter Truöl, my adviser, who initiated this project and was prepared at any time to stand by with his experience and experimental skill. Discussions with him always proved to be very helpful in overcoming problems that occurred during my work. He did much to make my graduate years most enjoyable.
- M. Schaad, L. van Elmbt, U. Straumann, M. Doser, J.P. Perroud, A. Bay, J. Imazato and T. Ishikawa for their collaboration in our research group.
- Stefan Vogel who assisted me in solving all sorts of mathematical and logical problems.
- The Physics Institute of the University of Zürich which offers the friendly and inspiring atmosphere which renders efficient work possible.
- The Swiss Institute for Nuclear Research (SIN) which provided all its experimental facilities with great generosity.
- R. Binggeli for her incessant moral support and her help in editing this draft.

This work was supported by the Swiss National Foundation.

Part I

SHORT VERSION

Chapter 1

MOTIVATION

Radiative muon capture (RMC) experiments on a wide range of nuclei are justified by the information they provide on the induced pseudoscalar branch of the semileptonic weak interaction.

The induced pseudoscalar coupling constant is predicted by PCAC (the hypothesis of partial conservation of the axialvector current) to be $g_P \approx 7g_A$, where g_A is the axialvector coupling constant. It has no influence on any observables in nuclear β -decay and electron-capture because of the small momentum transfer in these reactions. Thus one is left with studies on μ -capture processes to obtain information about its strength.

Several experimental efforts have been made during the last 15 years to measure normal μ -capture in hydrogen ($\mu^-p \rightarrow \nu_\mu n$) [Bar+ 81a], struggling with the difficulties of the enormously small capture rate, the high target purity required, the neutron detection and the complications due to molecular and atomic processes. The last five measurements gave a weighted average of $g_P = (7 \pm 1.5)g_A$ in quite good agreement with the PCAC prediction, but still these results lack satisfactory precision and confirmation.

Besides, the momentum transfer is fixed at the spacelike value $q^2 \approx -0.9m_\mu^2$ in ordinary μ -capture. In the radiative process ($\mu^-p \rightarrow n\nu_\mu\gamma$) q^2 ranges from $-0.9 m_\mu^2$ (for $E_\gamma = 0$) to $+0.9 m_\mu^2$ for the maximum photon energy, for which the OPE-amplitude (One Pion Exchange) and thus the influence of g_P is enhanced by a factor of ≈ 3.5 . Therefore studies of RMC are clearly justified.

Unfortunately the direct investigation of the elementary process on hydrogen has been ruled out until now by the extremely low branching ratio expected ($\approx 10^{-4} \div 10^{-5}$), which is another complication added to all those listed for normal μ -capture on hydrogen. Since the capture probability

in nuclei is proportional to Z_{eff}^4 (Z_{eff} is the effective nuclear charge) such experiments do become possible in heavier nuclei where the capture probability is enhanced by up to three orders of magnitude. However, the advantage of high capture rates is nearly balanced by the theoretical difficulties of calculating the nuclear final states and the modification of the elementary process in nuclear matter.

If, on the other hand, g_p is once fixed to a reliable experimental or theoretical value, RMC provides a means for checking nuclear model calculations, or, with a reliable model, for studying the possible modification of the induced pseudoscalar formfactor of the proton by the surrounding nuclear medium as suggested by [Eri 77], [Eri 86].

In any case, some thoroughly elaborated theoretical models (e.g. [CRS 80], [Chr 81], [CG 85]), have already been worked out and the present work would like to be a contribution to the experimental efforts on RMC, which started nearly 10 years ago. [Har + 77], [Fri + 82], [Fri + 85], [Has + 84].

Chapter 2

THEORY

2.1 The Induced Pseudoscalar Interaction

The weak interaction of leptons and quarks is generally known to be a current-current interaction of pure (V-A)-form. However, in the case of semileptonic interactions, the participating composite hadrons can no longer be treated as pointlike particles. Since one cannot simply account for their hadronic structure by a well understood theory of strong interaction, all terms allowed by Lorentz-invariance are added with unknown formfactors to the weak current. In this way a weak magnetic and a scalar term join the vector part and a tensor and pseudoscalar term join the axial vector part of the current. Assuming G-parity conservation, the tensor and scalar formfactors have to vanish and we are left with the magnetic and pseudoscalar formfactor (g_P), which we will concentrate on now.

2.2 The PCAC Prediction

As a consequence of the standard model of the electro-weak interaction the vector current is conserved (CVC). Assuming in addition a partial conservation of the axialvector current (PCAC) (i.e. the only source of axialvector current is the pion-current), a prediction for g_P becomes possible. By application of the Goldberger-Treiman relation [GT 58] the following simple relation between the axialvector and pseudoscalar formfactors for interactions of a lepton l with a nucleon is found:

$$g_P^l(q^2) = g_A \frac{2m_p}{m_\pi^2 - q^2} m_l \quad (2.1)$$

Applying this to the situation of muon capture on a proton, one gets:

$$g_P^\mu(q^2 \approx -0.9m_\mu^2) \approx 7g_A$$

It is usually assumed that the q^2 -dependance of g_P is merely $1/(m_\pi^2 - q^2)$ (pion pole dominance).

2.3 Measurement of g_P

Of all semileptonic processes μ -capture is the most promising for the determination of g_P . Because of the small momentum transfer no observables in either β -decay or electron-capture are reasonably sensitive to g_P . In the normal capture process ($\mu^- p \rightarrow \nu_\mu n$) the capture rate as well as the spin correlations of the neutron with the muon and the neutron helicity depend on g_P . In the case of radiative capture ($\mu^- p \rightarrow \nu_\mu n \gamma$) one finds three observables coupled with g_P : the photon yield, the photon polarization and the photon muon spin angular correlation.

2.4 The Photon Yield in Muon Capture

There is no major difficulty in calculating the differential photon- yield for RMC on a proton. Indeed, there even exist three different approaches to the problem which all lead to the same result. Fig. 2.1 shows the photon spectra obtained by [GO 81] for different values of g_P . It can easily be recognized that only the total photon yield, but not the shape of the energy spectrum is affected by g_P .

The elementary radiative capture process has a calculated rate of the order of $2 * 10^{-2} sec^{-1}$ while the muon decay rate is $4.6 * 10^5 sec^{-1}$. Thus, a muon stopped in a hydrogen target has an extremely low probability of about $4 * 10^{-8}$ to produce a high energy photon.

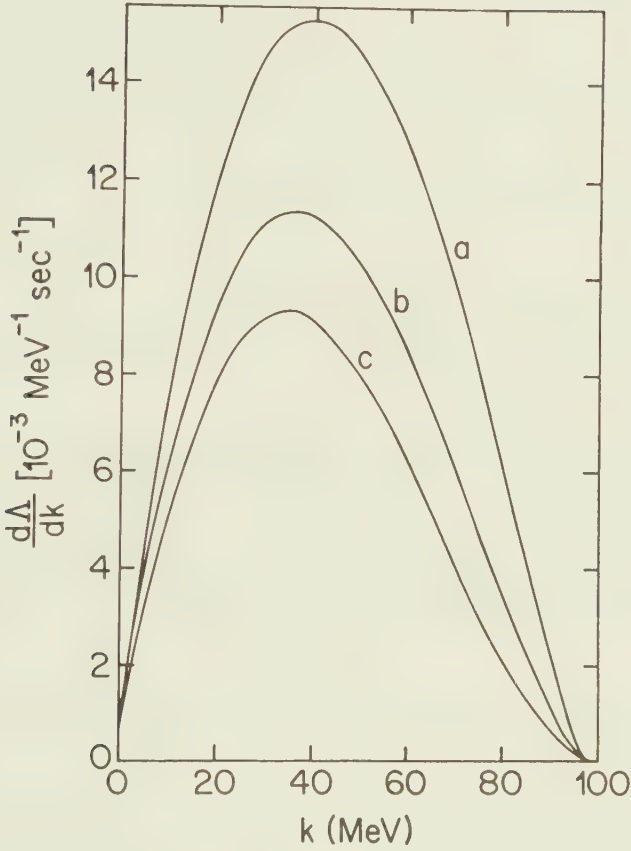


Figure 2.1: Photon yield for μ -capture on a proton as a function of the photon momentum k for different values of g_P , ([GO 81]). (a) $g_P = 14g_A$, (b) $g_P = 7g_A$, (c) $g_P = 0$.

2.5 Radiative Muon Capture in Nuclei

Because of the very low photon production rate experiments on RMC in hydrogen have not been possible until now. Two attempts will be made in the near future [Bad+ 86], [AH 86].

RMC experiments with heavier nuclei are feasible. Yet two severe problems enter the calculation of the photon yield:

- The elementary amplitude for the capture on one proton must be summed up over all protons in the nucleus.
- Amplitudes for all possible nuclear final states must be summed up with the appropriate weight.

There exist several approaches to these problems, which do not lead in all cases to quite the same outcome. More details about these calculations are to be found in chapter 9.

In the following, only those results are given which will be used for the analysis and interpretation of the experimental data.

2.6 Energy Spectra and Total Branching Ratios

- a) For ^{40}Ca there exist several calculations. In Fig. 2.2 a) the result of Christillin [Chr 81] ('phenomenological nuclear response') is presented. We will refer to it with 'CH'.
- b) For ^{16}O Christillin and Gmitro [CG 85] have established a theory applying a similar method, Fig. 2.2 b). It will be referred to by 'CG'.
- c) Fig. 2.2 c) shows the polynomial first used by Primakoff ([Pri 59], [RT 65]) which results if the nucleus is treated in the closure approximation with the assumption that only the muon radiates. It can be used for any nucleus ([CRS 80]) and depends only on the average nuclear excitation energy. We shall call it 'PP'. The polynomial itself is:

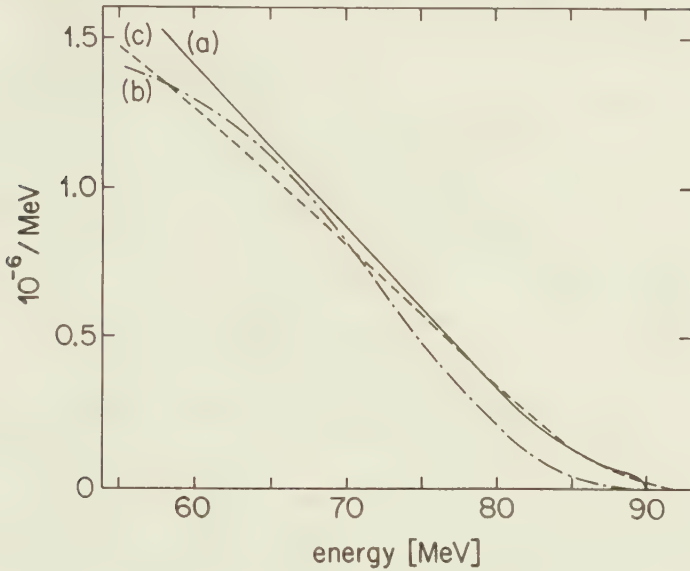


Figure 2.2: Theoretical RMC energy spectra for ^{40}Ca [Chr 81] a), for ^{16}O [CG 85] b), and a simple polynomial [Pri 59] c).

$$\frac{d\Lambda}{dk}(x) \sim (1 - 2x + 2x^2)x(1 - x^2), \quad x = \frac{k}{k^{max}} \quad (2.2)$$

k^{max} = max. photon energy.

Remarks:

- Only the spectra above 55 MeV are shown because the region below is experimentally not accessible, as will be explained in the next chapter.
- All these spectral shapes can be approximated nearly satisfactorily by a straight line, which is determined by the cut-off energy and the total photon yield only (we refer to it by 'SL').
- The RMC energy spectra are featureless and all the presented theoretical shapes do not differ much. Therefore, experiments are not very selective among the different theories, as a measurement with

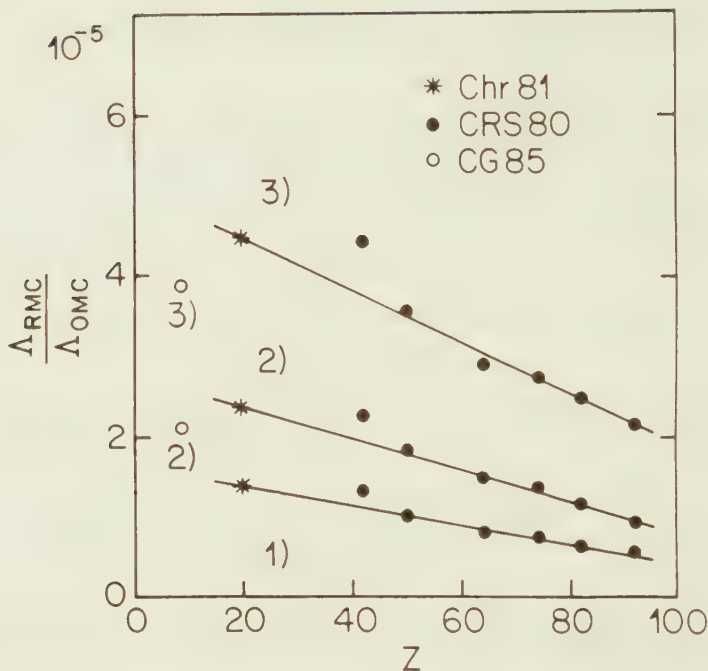


Figure 2.3: Predicted RMC branching ratios for photon energies > 57 MeV normalized to the total capture rate versus nuclear charge Z . 1.) $g_P = 0$, 2.) $g_P = 7$, 3.) $g_P = 14$.

a high resolution pair spectrometer has shown [Fri + 85]. Nevertheless, the total photon yield can be determined.

Figure 2.3 presents the total RMC branching ratios for photons with energies > 57 MeV (normalized to the total capture rate) for various nuclei. For $Z > 40$, the results of a calculation of Christillin [Chr 80] with a Fermi gas model and the closure approximation are shown.

Chapter 3

Experimental Set-up

3.1 The Muon Beam

The experiment was carried out in the μE1 zone of the SIN at Villigen (Fig. 3.1).

There, a π^- beam of 220 MeV/c momentum is injected into an 8 m long superconducting solenoid decay-channel (C)¹. At its end the muon beam is extracted (quadrupole doublet Q, bending dipole B) at a momentum of 125 MeV/c (backward decay muons, measured range width 4.5 g/cm², FWHM) and focussed onto an intensity slit system (S). This allows to vary the muon flux from 350 kHz up to 28 MHz (at a primary proton current of 140 μA). By a last quadrupole triplet the beam can finally be focussed onto the target with a spot size of ≈ 7 cm ϕ .

As Fig. 3.2 shows, the beam entered the experimental area via a first counter hodoscope (μ_1) of six 5 mm plastic scintillator strips with individual photomultipliers to reduce the counting rates. Through a second defining counter μ_2 (5 mm scintillator) the beam passed into a 10 cm long polyethylene degrader and then partially stopped in the target of 7×8.5 cm² and ≈ 5 g/cm² thickness (placed under 45° to the beam). The target was surrounded by a closed scintillator box (1 mm wall thickness). This box was divided into a front part (called μ_3) defining the entrance of a particle and into a rear part (denoted with μ_4) consisting of an open (five faced) cube which vetoed the particles passing through the target. Thus, a muon stop was indicated by $\mu_{st} = \mu_1 \wedge \mu_2 \wedge \mu_3 \wedge \bar{\mu}_4$. After determination of the stop rates per μ_1 the target box was replaced by a synthetic foam support (0.002 g/cm³) to avoid background events.

¹maximum field of 50 kG

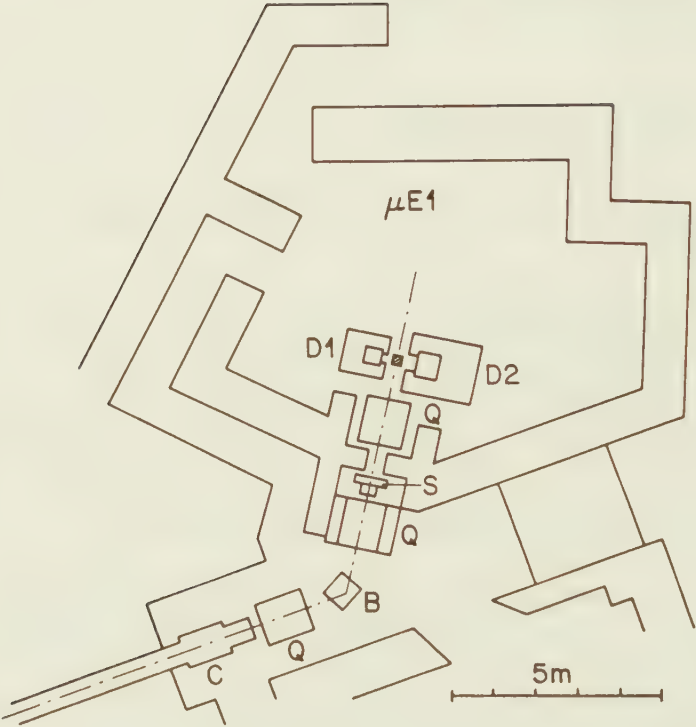


Figure 3.1: Experimental area $\mu E1$ at SIN

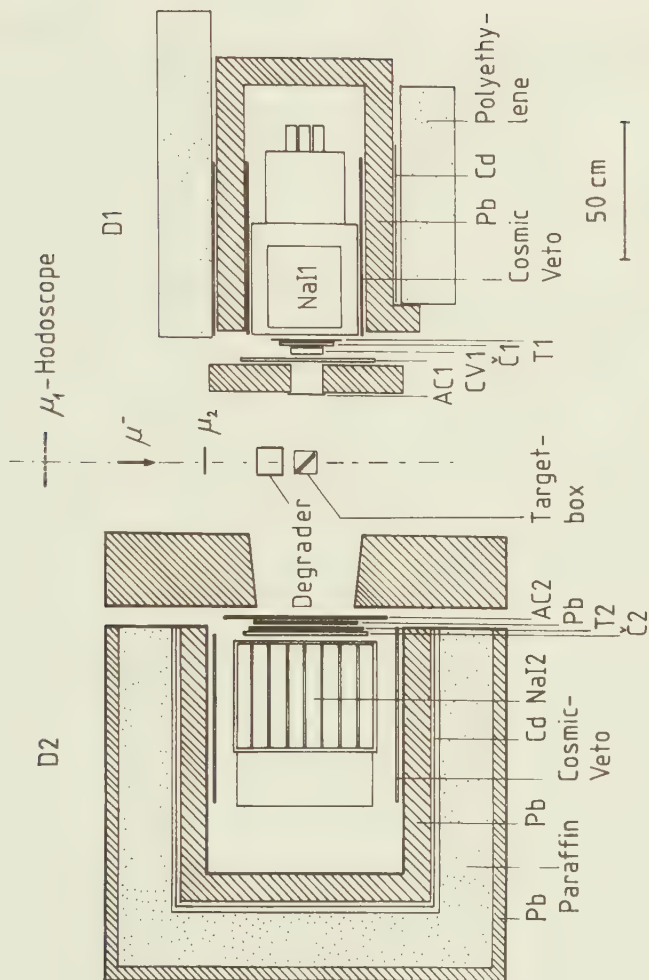


Figure 3.2: Experimental set-up

3.2 The Detectors

As shown in figures 3.1 and 3.2, two detector systems were placed perpendicularly to the beam. The first (D1) consisted of a single cylindrical NaI (27 cm ϕ , 30 cm length) (NaI1) with seven PM tubes. It was surrounded by four 55 x 60 x 1 cm³ cosmic veto-counters and shielded by a 10 cm lead, 2 mm cadmium and 20 cm polyethylene housing. It looked at the target through a 11 x 11 cm² lead collimator (10 cm thick) with a scintillation anti-counter (AC1) at each end to veto charged particles.

Behind the collimator a photon converter telescope was installed in order to distinguish photons from neutrons: A small cylindrical NAI-crystal (12 cm ϕ , 2 cm length) (CV) converted about a third of the high energy photons into electron-positron pairs. A 1 cm plastic Čerenkov counter (Č1) then discriminated these electrons from eventual recoil protons due to neutron scattering. In order to confirm the CV - Č1 coincidence a 2 mm scintillation trigger counter (T1) completed the telescope and provided a sharp timing signal. A photon trigger is thus released by an $\overline{AC1} \wedge CV \wedge \check{C1} \wedge T1 \wedge NaI1$ coincidence. The pulses of all mentioned counters (except the veto counters) were fed to charge sensitive ADC's. The total energy of an event was given by the sum of the CV- and NAI1-energies plus a correction for the energy loss in the further counters. In addition the time between the passage of a beam particle through μ_1 and the event was recorded.

The opposite detector (D2) consisted of a 8 x 8 matrix of 2.5 x 2.5 x 16 in³ NaI-bars with individual PM-tubes (NaI2, NaI wall). The shielding was quite similar to the one of D1 with the only difference that the polyethylene was replaced by paraffin surrounded by an additional 5 cm lead layer. The photon telescope of D2 did not include an active converter. The place of the small NaI-crystal was taken here by a 4 mm (≈ 1 rad. length) lead sheet. Consequently, a photon signal was given by $\overline{AC2} \wedge \check{C2} \wedge T2 \wedge NaI2$. Each NaI module had its own ADC and TDC. The pulseheight of C2 and T2 were recorded, too. The sum of the energy deposits in all hit bars provided the information on the photon energy which was then corrected by a Monte Carlo calculation for the energy loss in the lead converter. For details about this detector system refer to chapter 11.

Chapter 4

Detector Performance

4.1 Energy Calibration

4.1.1 Detector D1

The energy calibration of D1 included calibrations of the small (CV) and the big NaI-crystal (NaI1). The calibration of the converter was easily done by means of μ -decay electrons passing through it (the corresponding energy spectrum has been established with the EGS¹ Monte Carlo program). The big crystal was calibrated by means of the radiative pion capture (RPC) spectrum of hydrogen (tuning the muon channel to pions and using CH_2 - and C -targets).

The total energy of an event is given by the sum of the energy deposits in CV and NaI1 plus a correction for the energy loss in the other counters and the housings of the NaI detectors. The calibration result for the energy sum spectrum is shown in Fig. 4.1. The energy resolution at 130 MeV was 11% (FWHM) (phenomenological resolution function).

The energy calibration of D1 was surveyed and corrected periodically by means of the energy spectrum of cosmic muons crossing the detector. These events could be selected by putting the cosmic veto counters into coincidence with NaI1.

4.1.2 Detector D2

The procedure for the NaI-wall was slightly more complicated. At first the 64 NaI bars were individually calibrated with γ -sources (^{137}Cs , ^{88}Yt)

¹Electron Gamma Shower computer simulation developed at SLAC

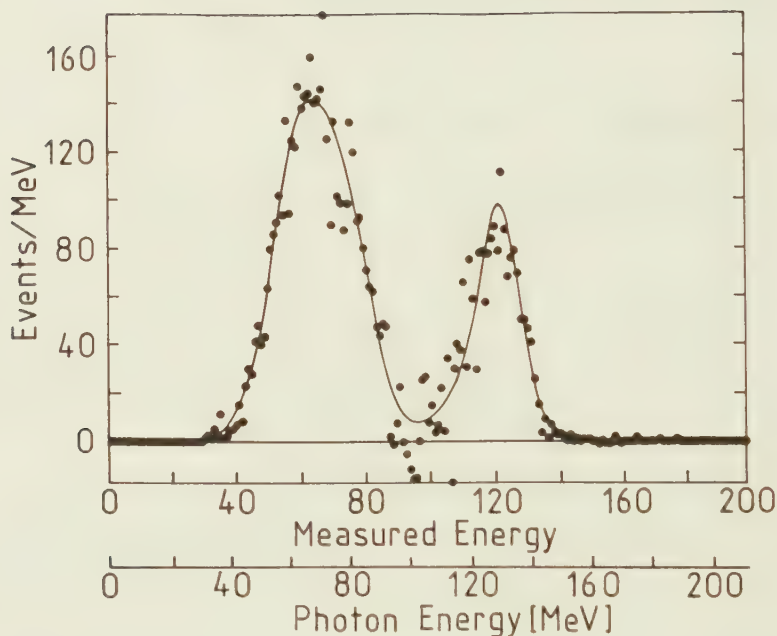


Figure 4.1: Photon energy spectrum of RPC on hydrogen. Result of a $C'H_2 - C'$ subtraction for detector D1. The solid line is a fit using the theoretical spectrum folded with a phenomenological resolution function. The lower scale is corrected for the mean energy loss in the converter telescope.

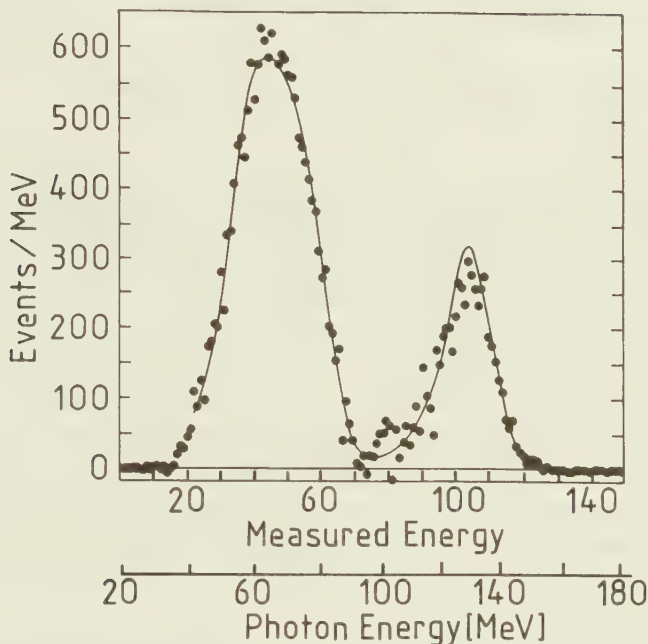


Figure 4.2: Photon energy spectrum of RPC on hydrogen for detector D2. The solid line represents the Monte Carlo simulation. The lower scale is corrected for the mean energy loss in the converter telescope.

and cosmic rays. To estimate the energy loss of the electron-positron pairs in the lead converter and in the Č2-counter, another EGS Monte Carlo simulation was used. It provided an estimate of the detectors response function to converted photons and a relation between the total energy deposited in all bars and the initial photon energy. The final energy calibration was established then by fitting the Monte Carlo spectrum for RPC on hydrogen with a (small) energy shift as a free parameter to the experimental data. The result of this fit is shown in figure 4.2. The energy resolution at 130 MeV corresponded to 14% (FWHM).

4.2 Acceptance Calibration

The acceptance calibration was performed in two steps.

- A first calculation established the *variation of the relative acceptance with energy* $r(E)$ (normalized to 1 at 130 MeV). For D1 it is given by the energy dependence of the photon conversion cross section [HUB 80]. The acceptance at low energies is modified by the fact that an electron-positron pair, after passing through the whole converter telescope, eventually cannot deposit enough energy in NaI1 to trigger it. This correction was measured with RPC photons from CH_2 by varying the NaI1 trigger threshold.

For D2 the variation of acceptance with energy was taken from the EGS-simulation. For RMC-measurements a further condition was imposed that at least 3 bars of the NaI wall must be hit by the photon shower in order to suppress neutron events. The reduction of acceptance caused by this condition was determined with RPC runs on a CH_2 target.

- The *absolute acceptances* were derived from known RPC-branching ratios [BCT 77], [Mor 79] [Per 79]. For D1 it was determined with a carbon target. For D2 the hydrogen RPC yield from the $CH_2 - C$ subtraction was used because the low energy tail of the carbon RPC spectrum was cut off by the NaI wall energy threshold. The absolute acceptances have been normalized to a photon energy of 130 MeV. The results are:

$$\underline{a_1(130 \text{ MeV}) = (1.18 \pm 0.09)10^{-3}} \quad (\text{for D1})$$

$$\underline{a_2(130 \text{ MeV}) = (3.80 \pm 0.33)10^{-3}} \quad (\text{for D2})$$

Figure 4.3 shows the results of the acceptance calibration for the two detectors, i.e. $a(E) = a(130 \text{ MeV}) r(E)$.

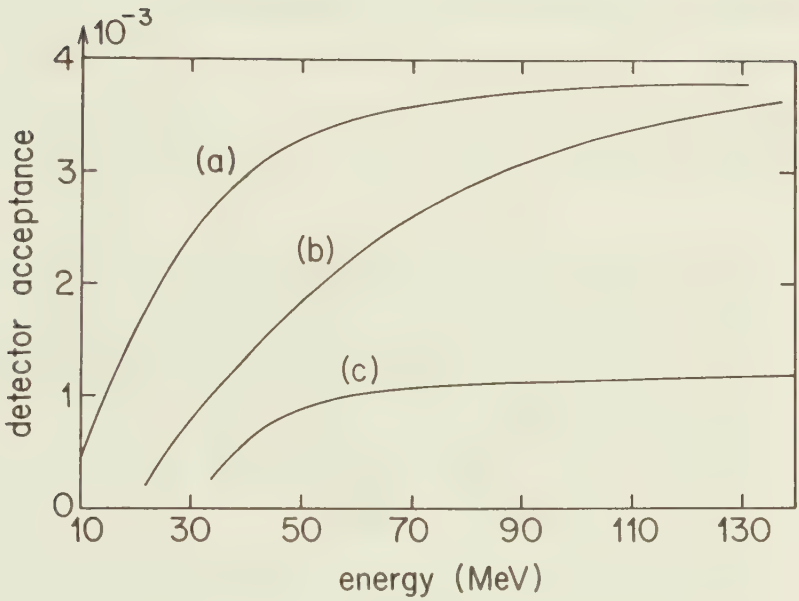


Figure 4.3: Acceptances $a(E)$ in function of energy. (a) for D2, (b) for D2 with at least 3 bars hit, (c) for D1.

Chapter 5

How to Tag RMC Photons

The low photon production rate is not the only problem appertaining to RMC experiments. There are four additional types of background troubling the measurements.

- An immense photon background from the **radiative decay** of the muon ($\mu^- \rightarrow \nu_\mu \bar{\nu}_e e^- \gamma$) and the **bremsstrahlung** produced in the target by the Michel-electrons arising from the normal muon decay completely hide the RMC spectrum below photon energies of about half the muon mass. For a muon bound at a nucleus, the bremsstrahlung energy spectra extend to slightly higher energies. They have been calculated [HVR 74], folded with the resolution functions of the two detector systems and fitted to the experimental spectra (figure 5.1).

This made it clear that with the poor resolution of our detectors, the RMC spectrum can only be disentangled from bremsstrahlung photons for energies > 64 MeV.

- The **neutron production** rate from ordinary muon capture (OMC) is ~ 100 times bigger than the photon production rate in the accessible RMC energy region. For D1 the neutron background could be reduced to an insignificant level by means of the active NaI converter. For D2, however, recoil protons from scattered neutrons in C^{12} produced a considerable background in the photon spectra by firing C^{12} with ‘knock-on’ electrons. The number of these neutron events could be reduced and determined with sufficient accuracy using the knowledge of the spatial energy distribution in the NaI wall (table 10.2).
- There were two sorts of **non target associated background**:

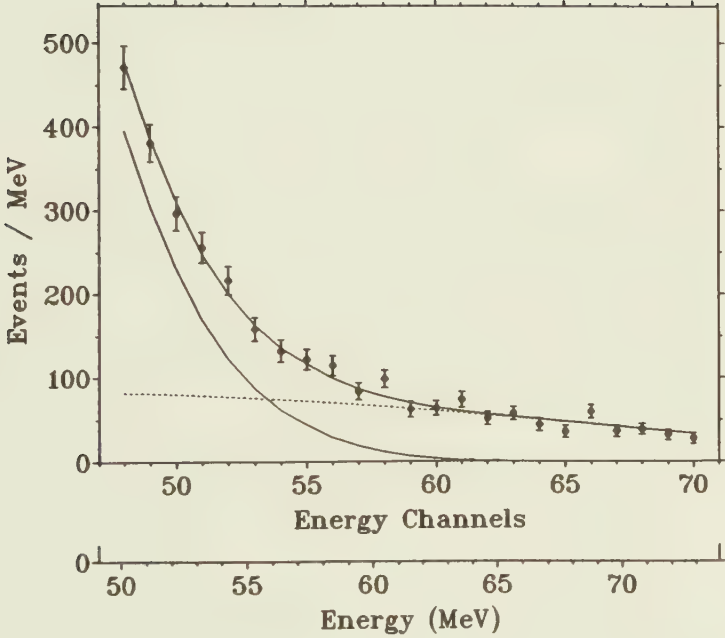


Figure 5.1: Photon spectrum from ^{nat}Fe -target from detector D1 fitted with a theoretical RMC spectrum of polynomial form (PP) and the simulated bremsstrahlung spectrum. The lower scale is corrected for the mean energy loss in the converter telescope.

First, beam particles stopped in the degrader produced photons accepted by the detectors. Their contribution to the total photon signal was $2 \div 25\%$ in the RMC' region (table 10.2). They were subtracted by means of empty target measurements.

Secondly, cosmic rays imitated the photon event pattern in detector D2. These events, too, disappeared almost completely with the empty target subtraction. The energy spectrum of cosmic events was measured with the accelerator switched off. The fraction of remaining cosmics in the RMC-spectra was then estimated by the number of events beyond 200 MeV, where only cosmic events were present.

- The μ E1 muon beam was contaminated with $\sim 0.02\%$ pions. Because of the large **RPC** branching ratios,¹ this resulted in a considerable photon background.

Yet, the RPC' events could easily be recognized by their time structure, since π^- capture is always prompt. Figure 5.2 shows the arrival time distribution of photons with respect to the signal of the first beam counter μ_1 .

A cut on nonprompt events removed $80 \div 90\%$ of the pion events (though not all, because of the μ_1 inefficiency). On the other hand, prompt events clearly show an enhanced RPC' energy spectrum above 100 MeV (figure 5.3). With a *subtraction of prompt spectra* from nonprompt spectra (which brings the number of events above 100 MeV to zero) the RPC' photons were removed. A factor S_0 accounts for the fraction of RMC' events oversubtracted by this procedure.

- Accidental coincidences of the light pulses from a bremsstrahlung photon and a decay electron in the NaI-crystals of D1 produced **pile-up events** in the RMC' energy region. This was fatal for the ^{12}C and ^{16}O measurements of D1 because of the high electron production rates of these light targets. For the heavier targets this contribution was relatively small (table 10.2) and was taken into account by a Monte Carlo simulation.

¹they are greater than the RMC branching ratios by a factor of $\approx 10^3$

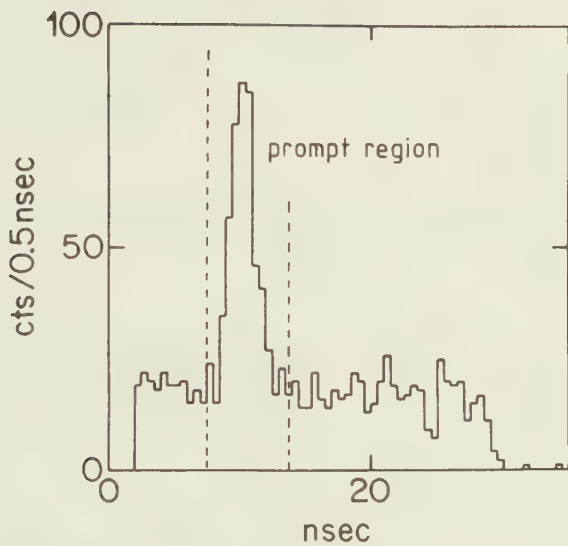


Figure 5.2: Distribution of time between the passage of a particle through μ_1 and a photon event in D1. The major part of all events is in the overflow of the spectrum.

A final formula to calculate the correct RMC energy spectra $N_\gamma(E)$ can be found in section 10.6.

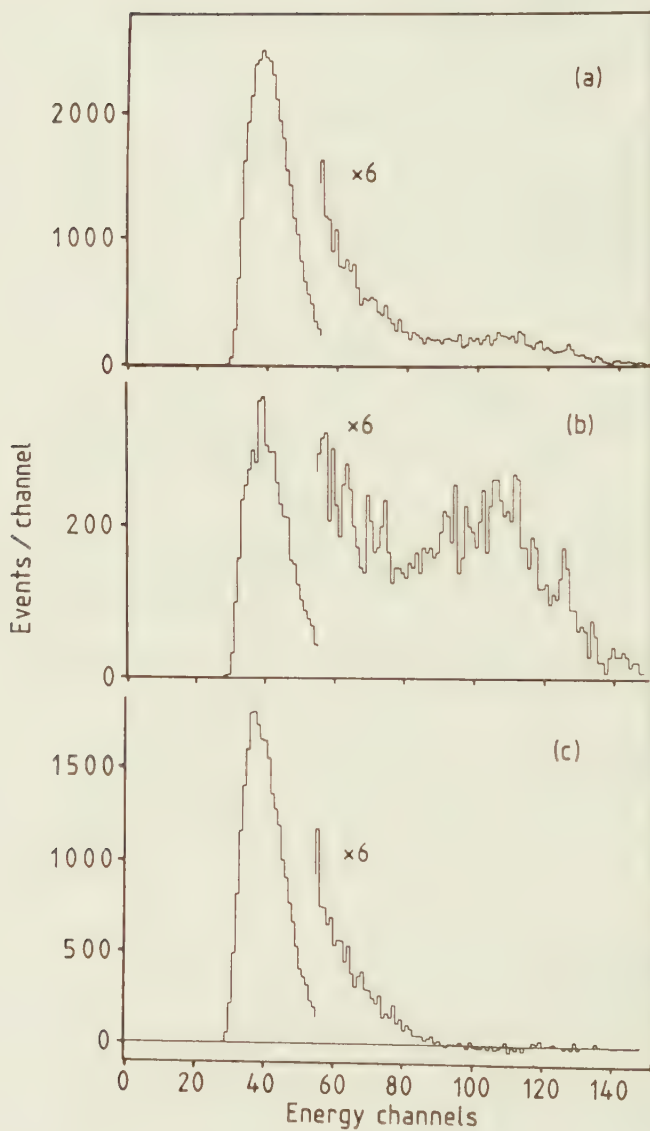


Figure 5.3: D1 Photon energy spectra from ^{40}Ca target. (a) nonprompt events, (b) prompt events, (c) after subtraction of prompt events.

Chapter 6

Calculation of Branching Ratios

6.1 The Formula

Let $T(E)$ be the theoretical RMC-photon energy spectrum (per captured muon) and $N_{tg}(ch)$ the measured photon spectrum from target in energy channels 'ch'. It is convenient to normalize $T(E)$ to 1 in an energy region ΔE , and to extract the RMC branching ratio $B_{RMC}(\Delta E)$ for photons in this energy region, thus writing:

$$T(E) = T_N(E)B_{RMC}(\Delta E).$$

where

$$\int_{\Delta E} T_N(E)dE = 1$$

The branching ratio $B_{RMC}(\Delta E)$ can consequently be fitted to the experimental data by the prescription:

$$N_{tg}(ch) = B_{RMC}(\Delta E) \int_{-\infty}^{\infty} r(E)T_N(E)R(E, ch)dE \quad (6.1)$$

$$\cdot \frac{\mu_{st}f_{cap}(1-f_{abs})(1-f_{veto})a(130MeV)}{S_O(1-f_n)(1-f_{pu})}$$

$r(E)$ is the relative acceptance of the detector

$R(E, ch)$ is the normalized resolution function of the detector

μ_{st} is the number of muon stops in the target

f_{cap} is the probability of a stopped muon being captured in the nucleus [ESW 66].

f_{abs} is the probability of a photon (of a mean energy in ΔE) being absorbed in the target itself

f_{veto} is the probability of a photon event being vetoed by a simultaneous Michel electron

$a(130 MeV)$ is the absolute detector acceptance at 130 MeV

S_O is the correction factor for the RMC' events being oversubtracted by the prompt event rejection

f_n is the overall fraction of neutron events still present in the spectrum

f_{pu} is the overall fraction of pile-up events

The energy intervall within which the experimental spectrum is fitted has not necessarily to be the same as ΔE . In the present case the channels corresponding to energies >64 MeV are taken into account for the fit, while ΔE is the energy region >57 MeV.

6.2 Fit Procedure

The experimental spectra are fitted by the sum of a simulated bremsstrahlung spectrum and the theoretical RMC' spectrum:

In a first χ^2 -fit including only the lower part of the RMC' region, with the amplitudes of bremsstrahlung and RMC' as free parameters, the actual 'endpoint' of the bremsstrahlung is determined.

In a second fit of the upper part of the RMC' region, the endpoint E_0 of the spectrum is established for those theories with variable E_0 . In this case E_0 and the RMC' yield are free parameters.

The RMC' branching ratio is then determined in a last fit including only the RMC' region fixed above. In order to base the results on the experimental data $B_{RMC'}(\Delta E)$ is finally multiplied by the ratio of the actual number of events in the considered energy region to

the integral of the fitted curve in the same region. (As it is known from the theory of χ^2 -fits this ratio is near to 1 anyway).

6.3 ^{12}C

For the ^{12}C target only very scarce data exist. They were not taken during the same run period as the data for all other targets.

Essentially, the analysis procedure was repeated for these data, though with two differences:

1. The whole Monte Carlo calculation for the resolution function of the detector had to be repeated, since a lead converter of 6 mm (instead of 4 mm) was used.

In consequence the mean energy loss in the converter changed, and the resolution of the detector deteriorated to 20% (FWHM) at 130 MeV (14% for 4 mm lead). The lower limit of the useful RMC region had therefore to be shifted up to about 70 MeV.

2. The absolute normalization of the photon yield was rather uncertain because of the unknown efficiency of the μ_3 -counter. For this reason the ^{12}C data were normalized to the ^{40}Ca data taken with high statistics under the same conditions, using the branching ratio measured during the second run period).

Chapter 7

Results

7.1 Example

As an **example** the plots of the analysed ^{nat}Fe data are shown. Figure 7.1 and 7.2 display the fitted data for D1 and D2 respectively. As shape of the theoretical RMC spectrum, the polynomial form (PP) was used.

Figure 7.3 illustrates the selectivity of the χ^2 -fit for theoretical RMC shapes and yields. The best fits of the PP and CH theories to the data are compared to the yields expected by [CRS 80] for $g_P = 0$ and $g_P = 7g_A$.

A direct comparison of the D1 and D2 data is given in figure 7.4. The photon spectra were shifted by the mean energy loss in the converter telescope, divided by the detector acceptance and normalized per μ_{st} (see section 13.1).

The corresponding graphs for the other targets are added in the appendix.

7.2 Compilation of Tables

Table 7.1 lists the useful energy regions for the χ^2 -fit of the branching ratios as well as the number of events falling into this interval.

The values of E_0 fitted to the data with PP and a straight line are given in table 7.2 for both detectors.

The fitted branching ratios for photons with energies higher than 57 MeV are collected in table 7.3.

The branching ratios for photons ≥ 64 MeV are given in the appendix.

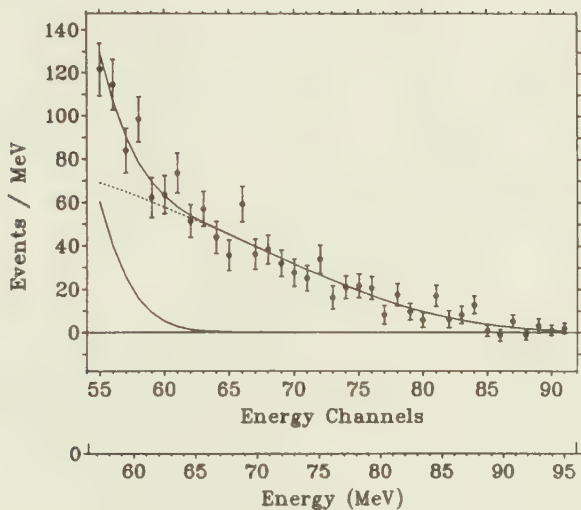


Figure 7.1: ^{nat}Fe spectrum for D1 fitted with PP. The lower scale is corrected for the mean energy loss in the converter telescope.

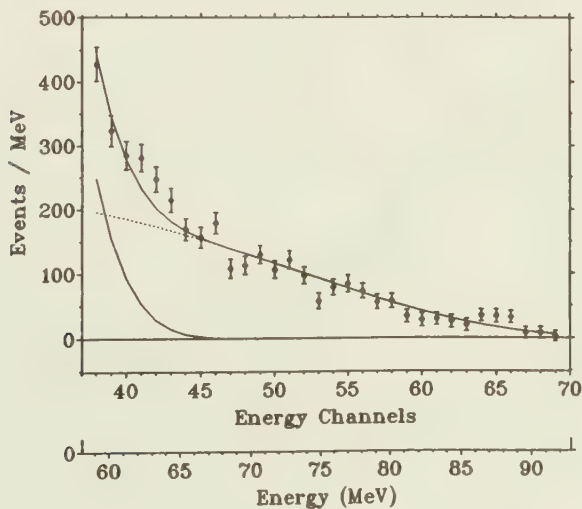


Figure 7.2: ^{nat}Fe spectrum for D2 fitted with PP

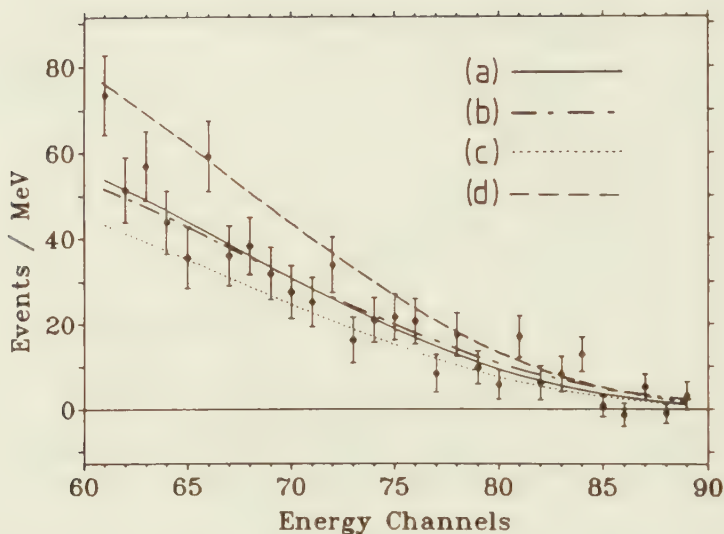


Figure 7.3: Comparison of theories with D1 ^{nat}Fe data. (a),(b): best fits for PP, CH. (c),(d): PP according to branching ratios of [CRS 80] for $g_P = 0$ and $g_P = 7g_A$.

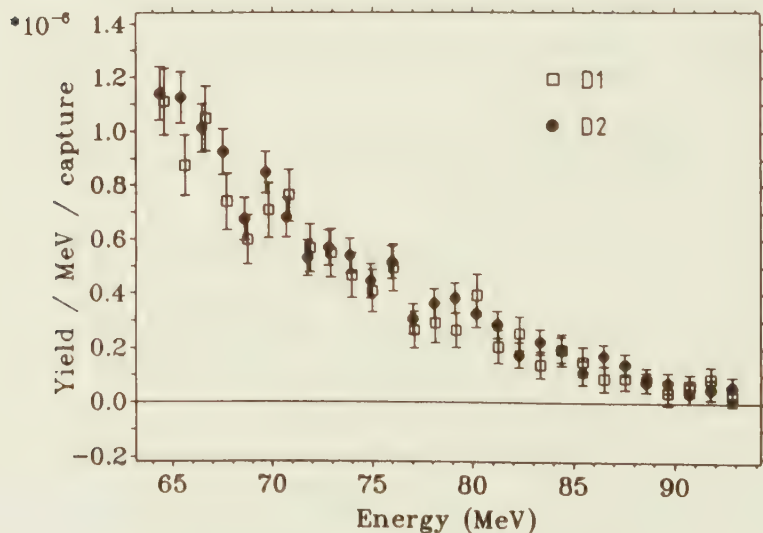


Figure 7.4: Comparison of 'unfolded' ^{40}Ca spectra for D1 and D2

Target	Detector D1		Detector D2	
	Fit Interval [MeV]	Number of Events	Fit Interval [MeV]	Number of Events
^{12}C	-	-	[71,88]	75 ± 46
^{16}O	[65,92]	420 ± 25	[65,90]	325 ± 37
^{27}Al	[64,93]	613 ± 29	[65,90]	889 ± 46
^{40}Ca	[65,93]	946 ± 36	[65,90]	2288 ± 64
^{nat}Fe	[65,92]	609 ± 29	[65,90]	1819 ± 60
^{165}Ho	[64,85]	124 ± 16	[65,85]	180 ± 39
^{209}Bi	[66,91]	275 ± 27	[65,85]	865 ± 68

Table 7.1: Fit intervals and photon statistics

Target	E_o [MeV] fitted with PP		E_o [MeV] fitted with SL	
	D1	D2	D1	D2
^{12}C	-	-	-	-
^{16}O	-	89.9 ± 5.0	-	85.5 ± 3.3
^{27}Al	86.8 ± 1.8	90.8 ± 1.8	81.3 ± 1.2	85.8 ± 1.5
^{40}Ca	92.1 ± 1.8	92.6 ± 0.8	86.5 ± 1.0	87.9 ± 0.9
^{nat}Fe	89.7 ± 1.7	90.6 ± 1.2	85.4 ± 1.3	86.3 ± 0.9
^{165}Ho	83.7 ± 5.3	85.1 ± 11.4	78.1 ± 3.2	73.5 ± 4.9
^{209}Bi	88.5 ± 3.6	84.2 ± 2.7	81.2 ± 2.3	77.6 ± 2.0

Table 7.2: Fitted endpoints of RMC spectra

Target	Applied Theory	Detector D1			Detector D2		
		Branching Ratio [10^{-5}]	E_0 [MeV]	χ^2	Branching Ratio [10^{-5}]	E_0 [MeV]	χ^2
^{12}C	CG	-	-	-	2.7 ± 1.8	-	0.89
^{16}O	CG	3.1	-	1.28	2.44 ± 0.47	-	0.93
	PP	-	-	-	2.39 ± 0.46	89.9	0.91
	SL	-	-	-	2.38 ± 0.46	85.5	0.97
^{27}Al	PP	$1.85^{+0.58}_{-0.25}$	86.8	0.75	$1.80^{+0.57}_{-0.25}$	90.8	1.25
	SL	$1.85^{+0.58}_{-0.25}$	83.1	0.80	$1.83^{+0.59}_{-0.26}$	85.8	1.33
^{40}Ca	CH	2.26 ± 0.30	-	0.80	2.34 ± 0.30	-	1.14
	PP	2.23 ± 0.29	92.1	0.87	2.29 ± 0.30	92.6	1.18
	SL	2.29 ± 0.30	86.5	0.86	2.30 ± 0.30	87.9	1.28
^{nat}Fe	PP	1.59 ± 0.22	89.7	1.13	1.87 ± 0.25	90.6	1.47
	SL	1.59 ± 0.22	85.4	1.18	1.88 ± 0.25	86.3	1.62
^{165}Ho	PP	0.91 ± 0.18	83.7	0.70	0.58 ± 0.19	85.1	0.78
	SL	0.95 ± 0.19	78.1	0.71	0.82 ± 0.27	73.5	0.80
^{209}Bi	PP	0.58 ± 0.10	88.5	0.83	0.68 ± 0.12	84.2	1.62
	SL	0.63 ± 0.11	81.2	0.82	0.75 ± 0.13	77.6	1.66

Table 7.3: RMC branching ratios per capture for photon energies ≥ 57 MeV; χ^2 per degree of freedom.

Target	Branching Ratio [10^{-5}]	g_P/g_A	Applied Theory
$^{12}_6C$	2.7 ± 1.8	9.5 ± 7.2	[CG 85]
$^{16}_8O$	2.44 ± 0.47	8.4 ± 1.9	[CG 85]
$^{27}_{13}Al$	$1.83^{+0.58}_{-0.25}$	$2.5^{+3.9}_{-1.7}$	[CRS 80]
$^{40}_{20}Ca$	2.30 ± 0.21	$6.3^{+1.0}_{-1.5}$	[Chr 81]
$^{nat}_{26}Fe$	1.71 ± 0.17	3.0 ± 1.3	[CRS 80]
$^{165}_{67}Ho$	0.75 ± 0.13	-0.5 ± 1.4	[CRS 80]
$^{209}_{83}Bi$	0.62 ± 0.08	0.2 ± 1.1	[CRS 80]

Table 7.4: Values for g_P . Weighted mean of D1 and D2.

Finally, the values for g_P are presented in table 7.4.

Tables 7.3 and 7.4 are illustrated by figures 7.5, 7.6 and 7.7.

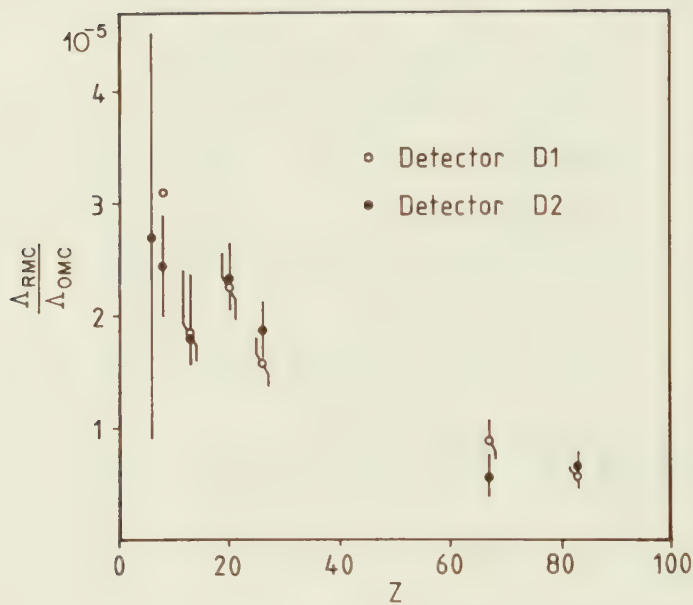


Figure 7.5: RMC branching ratios versus nuclear charge. Comparison of the two detectors.

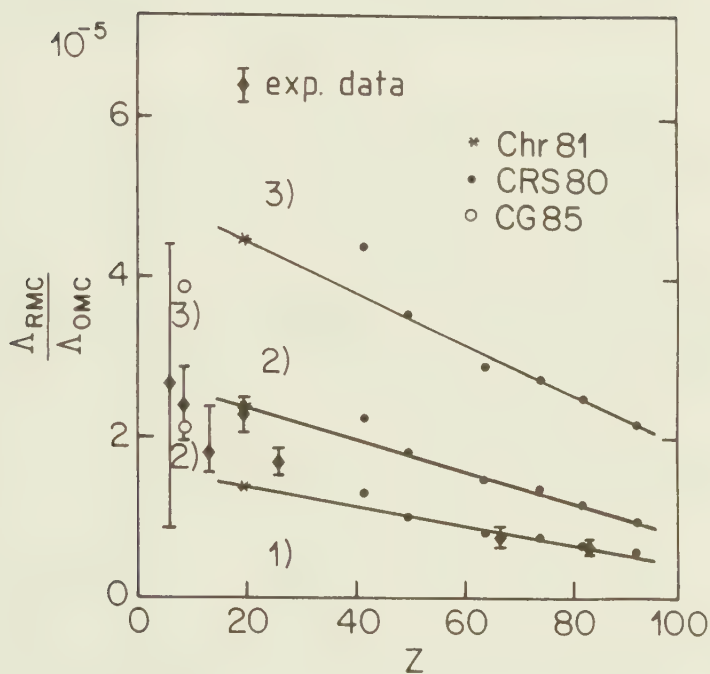


Figure 7.6: Mean value of the RMC branching ratios for the two detectors versus nuclear charge. Theoretical predictions : 1.) $g_P = 0$, 2.) $g_P = 7g_A$, 3.) $g_P = 14g_A$.

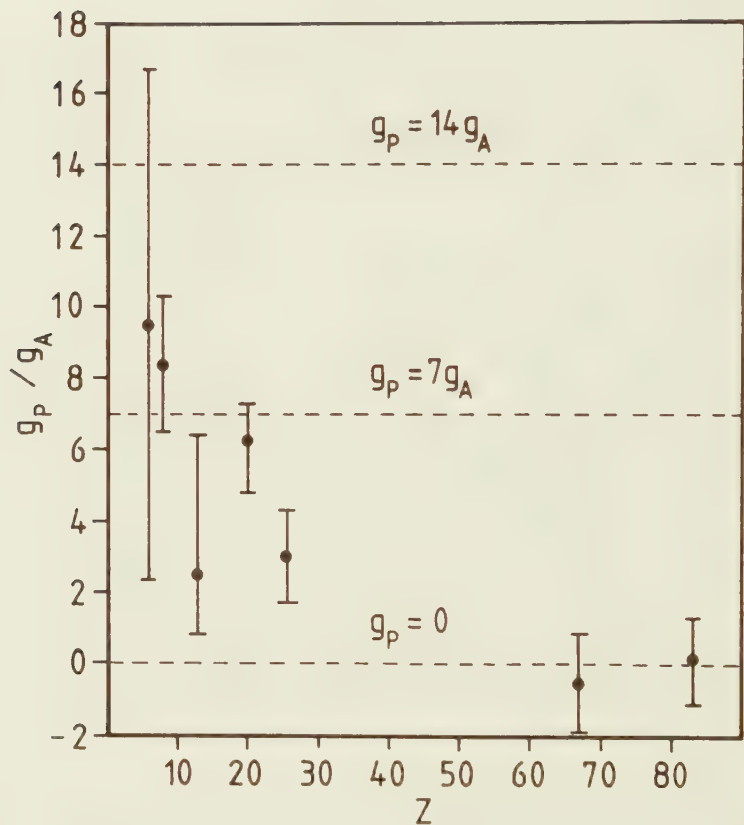


Figure 7.7: Induced pseudoscalar coupling constant g_p versus nuclear charge.

Chapter 8

Discussion and Conclusions

Figure 7.7 shows that the experimental results for g_P are consistent with the canonical PCAC value of $7g_A$ for low Z . For ^{12}C and ^{27}Al , however, they lack satisfactory precision. The theoretical predictions for the same isotopes are rather uncertain, too. In default of an appropriate calculation the theory for ^{16}O [CG 85] has been applied to ^{12}C . There does not exist any theory for ^{27}Al either. The predictions of the branching ratios for $Z \geq 40$ [CRS 80], including the value for ^{40}Ca [Chr 81] have therefore been extrapolated linearly to lower values of Z (figure 7.5).

The results for ^{nat}Fe , ^{165}Ho and ^{209}Bi indicate quenching of g_P . The experimental accuracy leaves no doubt about the discrepancy to the canonical value here. Since the applied Fermi gas calculation is fairly simple in comparison with the elaborated theories for ^{16}O and ^{40}Ca , the situation is still not conclusive. From this point of view the branching ratios for the heavy nuclei may be regarded as a test case for the nuclear model calculations. It would be most interesting to check if a thorough theory correctly accounting for the distribution of final states and including meson exchange currents (MEC) was able to reproduce the experimental branching ratios without renormalization of g_P . If this is not the case a strong indication of 'non classical' nuclear effects (section 9.7) is given.

Disregarding the uncertainties in the calculation of g_P from the measured branching ratios it is remarkable that the quenching of g_P seems to take place between atomic mass numbers of 20 to 60. [Gar 81] ([Wei 83]) found a suppression of g_A and g_M occurring in the same atomic mass region (by measurements of the Gamov-Teller strength in nuclear (p,n) reactions and magnetic isovector-spin transitions).

In section 9.7 a method is described to express a deviation of g_P/g_A by means of an effective mass of the virtual exchange pion in the capture process, which is equivalent to an effective polarizability α of nuclear matter. It is obvious that for ^{12}C , ^{16}O , and ^{40}Ca the polarizabilities calculated from the values of g_P presented here are consistent with zero since g_P agrees with the PCAC value. [DEF 77] predicts an α of -.75 for infinite nuclear matter. For the heavier isotopes the values derived for α are as follows:

$$^{56}\text{Fe} : \alpha = -.67 \pm .17$$

$$^{165}\text{Ho} : \alpha = -1.0 \pm .13$$

$$^{209}\text{Bi} : \alpha = -.98 \pm .10$$

Testing at last the reliability of the presented results, a comparison of the RMC branching ratio above 57 MeV for ^{40}Ca with the results of earlier measurements confirms full agreement with all existing data:

$$[\text{Har}+77] : B_{\text{RMC}} = (2.11 \pm 0.14)10^{-5}$$

$$[\text{Fri}+85] : B_{\text{RMC}} = (2.07 \pm 0.21)10^{-5}$$

$$\text{This work} : B_{\text{RMC}} = (2.30 \pm 0.21)10^{-5}$$

$$\text{Average} : B_{\text{RMC}} = (2.15 \pm 0.10)10^{-5}$$

Unfortunately the branching ratio for ^{16}O is in contradiction to a second value measured by [Fri+82]:

$$[\text{Fri}+82] : B_{\text{RMC}} = (6.2 \pm 0.8)10^{-5}$$

$$\text{This work} : B_{\text{RMC}} = (2.4 \pm 0.5)10^{-5}$$

A branching ratio of $\sim 6 * 10^{-5}$ would require values of $g_P > 14 g_A$ according to the theory of [CG 85].

Part II

DETAILS

Chapter 9

More on Theory

A complete review on RMC-theories and experiments is given in [GmT 87]. Further references are provided there.

9.1 General Form of Semileptonic Weak Interaction

The Hamiltonian of the current-current weak interaction can be written as:

$$H_W = \frac{G}{\sqrt{2}} J_\alpha^\dagger J^\alpha + h.c. \quad (9.1)$$

where the weak current J_α has a hadronic and a leptonic part:

$$J_\alpha = J_\alpha^h + J_\alpha^l \quad (9.2)$$

The lepton current can be represented as:

$$J_\alpha^l = \sum_{\text{lepton flavours}} \bar{\nu}_l \gamma_\alpha (1 - \gamma_5) l \quad (9.3)$$

where l represents the lepton field and ν_l the corresponding neutrino field (γ_α are the Dirac matrices). This expresses the (V-A)-form of the interaction.

Assuming that the strong interaction could be switched off, the hadronic weak current can be written as:

$$J_\alpha^h = (u, c, t) \gamma_\alpha (1 - \gamma_5) [KM] (d, s, b)^T \quad (9.4)$$

where u, d, c, s, t, b are the quarks fields and $[KM]$, the Kobayashi-Maskawa matrix, represents the weak flavour mixing.

If we exclude strange currents and restrict ourselves to nucleons we get:

$$J_\alpha^h = \bar{\psi}_p \gamma_\alpha (1 - \gamma_5) \psi_n \cos \theta_C \quad (9.5)$$

(θ_C is the Cabibbo-angle).

If the strong interaction is now turned on we have to account for the hadronic structure of the nucleons. Keeping the (V-A)-form of the interaction we write:

$$J_\alpha^h = \bar{\psi}_p (V_\alpha - A_\alpha) \psi_n \cos \theta_C \quad (9.6)$$

where V_α and A_α are the most general vector- resp. axialvector-terms allowed by Lorentz invariance:

$$V_\alpha = g_V(q^2) \gamma_\alpha + \frac{i}{2m_N} g_M(q^2) \sigma_{\alpha\beta} \frac{q^\beta}{2m_N} + g_S(q^2) \frac{q_\alpha}{2m_N} \quad (9.7)$$

$$A_\alpha = g_A(q^2) \gamma_\alpha \gamma_5 + g_P(q^2) \gamma_5 \frac{q_\alpha}{m_N} + i g_T(q^2) \sigma_{\alpha\beta} \gamma_5 \frac{q^\beta}{2m_N} \quad (9.8)$$

m_N is the nucleon mass and q_α the momentum transfer.

These terms contain now several formfactors induced by the strong interaction:

g_V	vector coupling
g_M	weak magnetism
g_S	scalar coupling
g_A	axialvector coupling
g_P	pseudoscalar coupling
g_T	tensor coupling

9.2 G-Parity

The semileptonic weak currents, which have become rather complex, can be simplified by means of G-parity conservation. G-parity is defined as:

$$G \equiv C e^{i\pi T_2} \quad (9.9)$$

which means charge conjugation with a rotation of 180° around the 2-axis in the isospin space.

Since strong interaction is invariant under G-parity the hypothesis has been established that all terms in V_α resp. in A_α should have a common G-transformation property.

Thus, we can *exclude the scalar and the tensor term as 'second class currents'* because they transform in opposite manner to the generic vector- resp. axialvector-term.

9.3 PCAC and the Goldberger-Treiman Relation

Considering only '**first class currents**' we may write the axialvector matrix element of a weak nucleon transition as:

$$\langle p | A_\alpha | n \rangle = \bar{\psi}_p [g_A(q^2)\gamma_\alpha\gamma^5 + g_P(q^2)\frac{q_\alpha\gamma^5}{m_N}] \psi_n \cos\theta_C \quad (9.10)$$

To illustrate the role of the formfactors $g_A(q^2)$ and $g_P(q^2)$ we interpret $\langle p | A_\alpha | n \rangle$ using the following three diagrams:

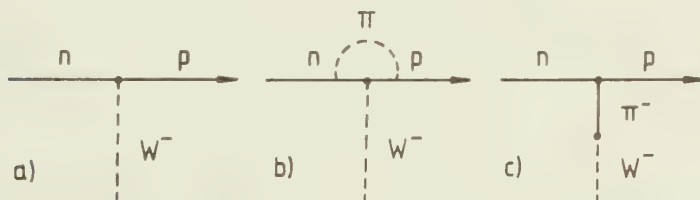


Diagram a) represents the transition of nucleons without hadronic structure and gives a contribution of 1 to g_A . Diagram b) accounts for the strong vertex corrections which modify g_A to be ≈ 1.25 . Finally the '*pion-pole*'-diagram c) shows the main contribution to the pseudoscalar interaction, thus to g_P . It corresponds to the amplitude:

$$a = (n \rightarrow p\pi^- \text{ vertex}) * (\pi^- \text{ propagator}) * (\pi^- \text{ decay})$$

Now the $(n \rightarrow p\pi^-)$ -vertex is described by the physical pion-nucleon coupling constant g_0 and the π^- -decay by the pion decay constant f_π . Resuming this we write:

$$a = -\cos\theta_C \frac{G_F}{\sqrt{2}} g_0 f_\pi \sqrt{2} \frac{1}{q^2 - m_\pi^2} \bar{u}_p q_\alpha \gamma^5 u_n L^\alpha \quad (9.11)$$

where L^α represents the lepton current from π -decay.

As compared to equation 9.10, we find:

$$g_P(q^2) = -f_\pi g_0 \sqrt{2} \frac{1}{q^2 - m_\pi^2} \quad (9.12)$$

Here we have neglected the vertex-corrections appertaining to diagram c) which would contribute another factor $F(q^2)$ to $g_P(q^2)$.

We assume that $F(q^2)$ is slowly varying with q^2 and

$$F(q^2 = 0) \approx F(q^2 = m_\pi^2) \approx 1.$$

Inserting now 9.12 into 9.10, the expression for the axialvector matrix element presents itself as follows:

$$\langle p | A_\alpha | n \rangle = \bar{\psi}_p [g_A(q^2) \gamma_\alpha \gamma^5 - \frac{f_\pi g_0 \sqrt{2}}{q^2 - m_\pi^2} \frac{q_\alpha \gamma^5}{m_N}] \psi_n \cos\theta \quad (9.13)$$

Finally we use the **PCAC** (*Partially Conserved Axialvector Current*) hypothesis which states that the axialvector-current is conserved for massless pions:

$$\lim_{m_\pi \rightarrow 0} \frac{\partial A^\alpha}{\partial x^\alpha} = 0 \quad (PCAC) \quad (9.14)$$

Applying this to equation 9.13 we are readily led to:

$$g_A(q^2 = 0) = f_\pi g_0 \frac{1}{\sqrt{2} M_N} \quad (9.15)$$

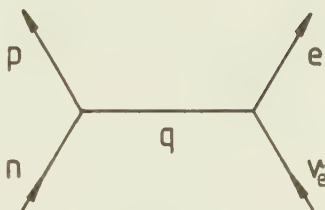
well known as the Goldberger-Treiman relation [GT 58]. With 9.12 it reads:

$$g_P(q^2) = g_A \frac{2m_N}{m_\pi^2 - q^2} \quad (9.16)$$

(For further details cf. [CB 83])

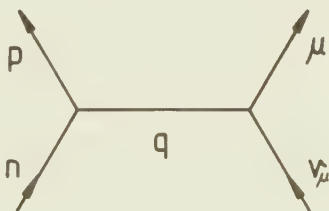
9.4 Testing PCAC

9.4.1 β -Decay



The momentum transfer $q^2 \approx (m_p - m_n)^2$ is very small and equation 9.10 shows that the pseudoscalar term is proportional to q_α/M_N , g_P thus having no effect on β -decay.

9.4.2 μ -Capture



For ordinary muon capture (OMC) the momentum transfer $q^2 \approx -0.9m_\mu^2$ is big enough and the pseudoscalar term in 9.10 becomes comparable to the axialvector term and 9.16 yields the prediction for g_P :

$$g_P(q^2 \approx -0.9m_\mu^2) \approx 7g_A \quad (9.17)$$

9.4.3 Radiative μ -Capture

If we have an additional photon, the momentum transfer is not fixed anymore. There are two possibilities: either the lepton or one of the hadrons radiates. Figure 9.1 a) shows the accessible regions for q^2 for the two cases [KLI 82]. Compared to ordinary capture, $g_P(q^2)$ is enhanced for the radiative capture due to its $1/(m_\pi^2 - q^2)$ dependance, especially for

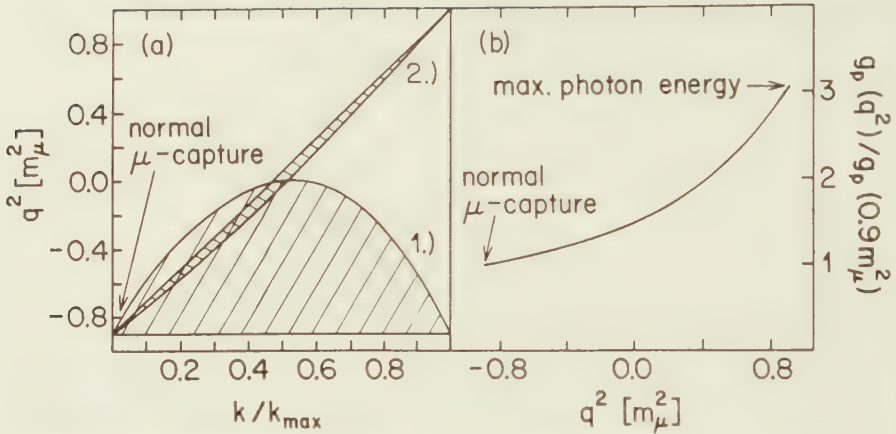


Figure 9.1: a) Accessible q^2 regions for RMC. 1.) The muon radiates. 2.) A hadron radiates. b) $g_P(q^2)$ normalized to its value for OMC.

positive q^2 -values being attained for high photon energies. Figure 9.1 b) shows the value of $g_P(q^2)$ for all possible q^2 compared to $g_P(q^2 = -0.9m_\mu^2)$.

9.5 Radiative Capture on the Proton

There are several theoretical derivations for the free-nucleon RMC amplitude ('*elementary amplitude*') and the respective results only differ by small higher order terms. Three of them are shortly described here:

- In the '*diagrammatic treatment*' [RT 65], photons are attached to all outer and inner lines of the capture diagram carrying charge or magnetic moment. The resulting graphs shown in figure 9.2 (from [RT 65]) are then treated as normal Feynman-diagrams.
- Adler and Dothan [AD 66] and later Christillin and Servadio [CS 77] found a solution to the problem by means of *low energy theorems* using the knowledge of current divergences [Low 58].
- Gmitro and Ovchinnikova [GO 81] completed a calculation of the elementary amplitude initiated by [HP 78], [WS 79] [Fea 80] applying the *conservation laws* of the electromagnetic and the weak vector current and PCAC.

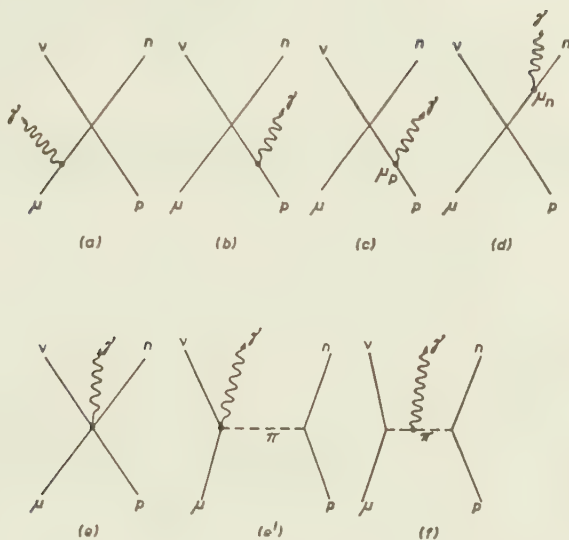


Figure 9.2: Graphs contributing to RMC [RT 65].

The result of this calculation has been depicted in figure 2.1.

The distribution of RMC-photons in function of the photon momentum k and the emission angle in respect to the muon spin can be expressed as:

$$\frac{d\Lambda_\gamma(k, \theta_{\mu\gamma})}{dk d(\cos\theta_{\mu\gamma})} = \frac{d\Lambda_\gamma(k)}{dk} (1 + \Gamma_\gamma(k) \cos\theta_{\mu\gamma}) \quad (9.18)$$

where the photon yield $d\Lambda_\gamma(k)/dk$ as well as the asymmetry parameter $\Gamma_\gamma(k)$ depend on g_P . As a third probe, the photon polarization is sensitive to g_P .

9.6 Radiative Capture in Nuclei

As already mentioned, RMC-spectra are only accessible for nuclei with $Z \geq 5$. The capture rate Λ_{cap} for a nucleus is proportional to Z^4 as long as there are no screening effects of the atomic electrons [BS 73]. Figure 9.3 shows the ratio of the number of captured muons to the total number of

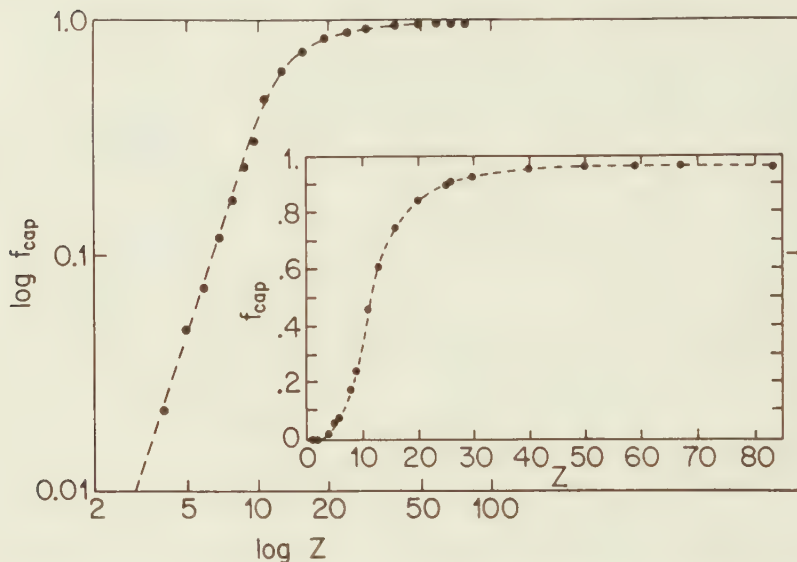


Figure 9.3: Capture probability in function of nuclear charge in logarithmic and linear scale [ESW 66].

muons stopped in a material with nuclear charge Z : $f_{\text{cap}} = \Lambda_{\text{cap}} \tau$ ($\tau = \mu^-$ lifetime in the material).

Calculating the RMC-amplitude for a nucleus, one is confronted with essentially two new problems:

- a) One has to **sum up** the elementary amplitude **over all protons** in the nucleus. There exist two basically different methods of carrying this out:

- **Impulse approximation (IA):**

It is assumed that only one proton is essentially involved in the process. Hence the RMC-matrix element between the initial state $|\psi_i\rangle$ and the final state $|\psi_f\rangle$ becomes:

$$M_{fi} = \langle \psi_i | H_{eff} | \psi_f \rangle \rightarrow \langle \psi_f | \sum_n H_n | \psi_i \rangle \quad (9.19)$$

where H_n denotes the nonrelativistic reduction of the RMC transition operator for one quasi-free proton which, summed up correctly

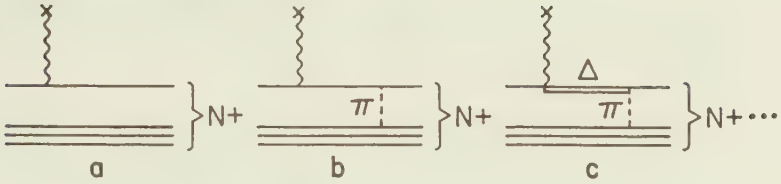


Figure 9.4: Examples of meson exchange currents

over all protons, leads to the effective operator H_{eff} .

The naive IA neglects interactions involving more than one nucleon (e.g. scattering of the exchange-pion). Figure 9.4 shows examples of so called *meson exchange currents* (MEC) which are taken into account in modified versions of the IA. [ATE 86], [GKO 84].

- **Elementary particle treatment (EPT):**

The elementary amplitude is directly calculated for the nucleus itself which is interpreted as an elementary particle with simple qualities such as spin, isospin, parity, etc.. The substructure of the nucleus is consequently described by a series of formfactors.

The major problem however consists in fixing the unknown formfactors. Some of them can be determined experimentally (e.g. the electromagnetic form factors from π -capture). Others can be calculated from the nucleon formfactors which again requires a nuclear model and methods such as IA. [KLI 82].

b) Since measurements of RMC to exclusive nuclear final states do not exist, one has to **sum up** the RMC-amplitudes **over all possible final states**:

- The **closure approximation (CA)** is the most simple and most often used method of carrying out this sum [RT 65] [RYY 74]. It assumes that all existing nuclear final states can be reached and that the maximum photon energy is described by a common mean excitation energy $\overline{\Delta E}$:

$$k^{max} = m_\mu - \overline{\Delta E}, \quad \forall |\psi_f\rangle \quad (9.20)$$

Since, for high photon energies, high lying final states cannot contribute [Chr 81], CA-calculations normally overestimate the RMC-rate.

- A more sophisticated technique was applied first by Foldy and Walecka [FW 64] to OMC-calculations and was used in RMC-calculation by Christillin [Chr 81] for ^{40}Ca and by Christillin and Gmitro [CG 85] for ^{16}O .

It is based on a **phenomenological response function** of the nucleus (e.g. for ^{16}O the giant dipole and quadrupole resonances and the discrete states $0^-, 1^-, 2^-, 3^-$ in ^{16}N are regarded). The individual transition strengths can be adjusted by using OMC- results.

Figure 2.3 shows a compilation of theoretical results for the total RMC yield for several models and values of g_P .

9.7 Renormalization of g_P

Some experimental facts such as the **EMC effect**, the suppression of the longitudinal response in ee' -**scattering** and the reduction of the **Gamov-Teller strength** in nuclear (p,n) reactions may be interpreted as a modification of the nucleon formfactors by the nuclear medium [Eri 86], [Wei 83].

It is not clear if this should be regarded as a *QCD effect* (quark-gluon plasma) or just as an extension of '*classical nuclear physics*' (mesonic and Δ degrees of freedom), meaning that the effects could be explained by a complete and consequent application of corrections such as MEC (see page 55).

[DEF 76] and [Eri 77] suggest that the effective values of g_A and g_P in a nucleus provide information on the polarizability of the nuclear matter through an effective mass term in the pion propagator:

$$m_{\pi'}^2 = m_{\pi}^2(1 + \alpha)^{-1} \quad (9.21)$$

α = axial polarizability.

The Goldberger-Treiman relation (eq. 9.16) yields readily

$$\frac{[g_P/g_A]^{eff}}{[g_P/g_A]^{free}} = \frac{m_\pi^2 - q^2}{m_{\pi'}^2 - q^2} \quad (9.22)$$

This allows to express a possible renormalization of the formfactors in simple terms without affecting PCAC.

Chapter 10

More on Background Events

10.1 Bremsstrahlung

A huge photon background arises from the radiative decay of the muon ($\mu^- \rightarrow \bar{\nu}_e \nu_\mu e^- \gamma$) and the bremsstrahlung produced in the target by ordinary decay-electrons. For a free muon, this photon spectrum cuts off at half the muon mass, but for a bound muon it extends to slightly higher energies.

As [FrT 82] pointed out, the energy spectra of inner and outer bremsstrahlung are quite similar and the latter dominates completely for targets of about half a radiation length. A Monte Carlo calculation as well as an analytic formula originating from Tsai [Tsa 74] gave nearly the same results for the energy spectra of bremsstrahlung leaving the different targets used. Input to the bremsstrahlung calculation were the decay electron spectra as provided by [HVR 74] for bound muons. Figure 10.1 shows the result for the $5.6 \text{ g/cm}^2 \text{ nat Fe}$ target.

Although the bremsstrahlung background ends at $55 \div 57 \text{ MeV}$ for all targets, a clean RMC signal can only be seen above $\approx 64 \text{ MeV}$ because of the fairly bad resolution of the detector system. (Fig. 5.1)

10.2 Neutron Background

The total RMC-photon yield between 57 and 90 MeV is of the order of 10^{-5} per μ -capture while the neutron yield from OMC (Ordinary Muon Capture) in the same energy region is ≈ 100 times bigger [Koz + 85]. Therefore, one needs a fairly efficient photon-neutron discrimination. In fact the photon conversion telescopes used for the two detectors do not

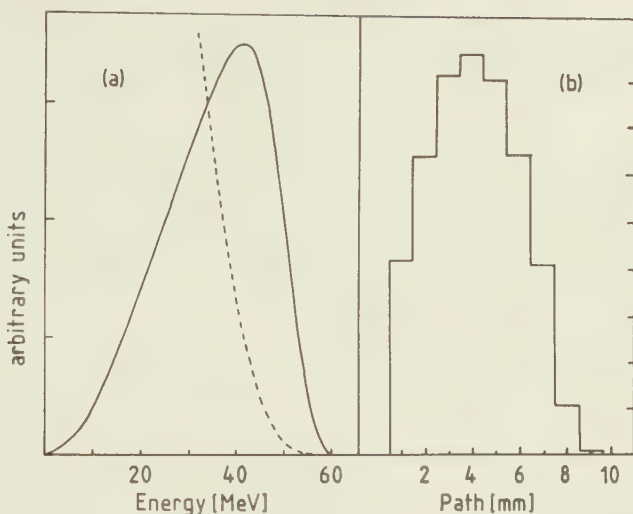


Figure 10.1: (a) Michel spectrum from muons bound in ^{nat}Fe (solid line) and smoothed Monte Carlo spectrum for the resulting bremsstrahlung (dashed). (b) Distribution of electron pathlengths through target.

absolutely exclude neutron events. It is still possible that recoil protons from neutrons (scattered in the converter NaI resp. in $\check{\text{C}}2$) knock on electrons that can produce light in the Čerenkov counters. This applies to protons of energies greater than 50 MeV [Lez 76]. Besides, there are other luminiscent processes by which any (even slow) charged particle produces light (though very little) in lucite material [Mad 56].

Measurements with pions stopped in a carbon target revealed that for D1 the fraction of neutrons (in the RMC energy region) undergoing the above mentioned process is $<0.004\%$ (only an upper limit could be estimated).

Yet, for D2 the neutron detection probability has been determined to $\approx 0.04\%$. This already leads to a contribution of $16 \div 40\%$ of neutrons (depending on the target) to the RMC photon signal. The knowledge of the **spatial density of the deposited energy** in the NaI matrix, however, allows to determine the fraction of neutrons in a photon spectrum and to reduce the neutron sensitivity of the detector by about a factor of 3.

Figure 10.2 a) shows the distribution of the number of bars hit by the

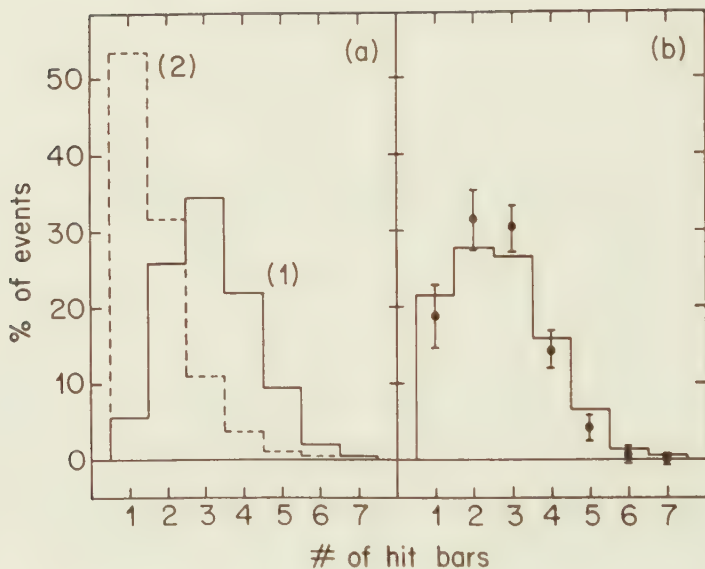


Figure 10.2: (a): Hit bars distribution for 50 to 60 MeV photons (1) and neutrons (2). (b): Hit bars for μ -capture events of the same energy from ^{165}Ho . The solid line is a fit with a mixture of photons and neutrons.

shower of photons of 60 to 70 MeV resp. of neutron events of the same energy. (Photons and neutrons were produced by pion capture in a CH₂ target and selected by their time of flight).

The neutron problem was finally settled in the following way: The RMC energy spectra were divided in 3 slices of 10 MeV width. Then the distributions of the number of hit bars of the events in the individual energy intervals ΔE were fitted with a mixture of the correspondent photon- and neutron- distributions (shower size depends on energy), according to:

$$m(\Delta E_i) = r_n(\Delta E_i)n(\Delta E_i) + (1 - r_n(\Delta E_i))\gamma(\Delta E_i), \quad (i = 1, 3) \quad (10.1)$$

where $m(\Delta E_i)$, $n(\Delta E_i)$, $\gamma(\Delta E_i)$ are the hit bars distributions of the unknown mixture, the pure neutrons and the pure photons, and $r_n(\Delta E_i)$ is the unknown fraction of neutrons in the spectrum (i.e. the free fit parameter) (Fig. 10.2 (b)).

Since the neutron fractions were considerable for some targets, the condition that the number of hit bars must be at least 3 was imposed on all RMC events. As figures 10.2 and 10.3 prove, a large share of the neutron events is removed in this way, while photons in the RMC energy region are not so much affected.

This again has been determined with neutrons from π -capture in CH₂.

If $a_n(\Delta E_i)$ denotes the remaining neutron fraction for the interval ΔE_i , the number of neutrons left in the total RMC spectrum is given by

$$N_n = \sum_{i=1}^3 a_n(\Delta E_i) r_n(\Delta E_i) N_m(\Delta E_i) \quad (10.2)$$

where $N_m(\Delta E_i)$ is the number of events measured without any condition on the number of hit bars.

For the calculations of the final branching ratios, it is convenient to define a factor f_n as the overall fraction of neutron events in the signal from target. Because of the energy dependence of the electromagnetic shower size it is not possible to remove the remaining neutrons by a subtraction of events with less than 3 hit bars.

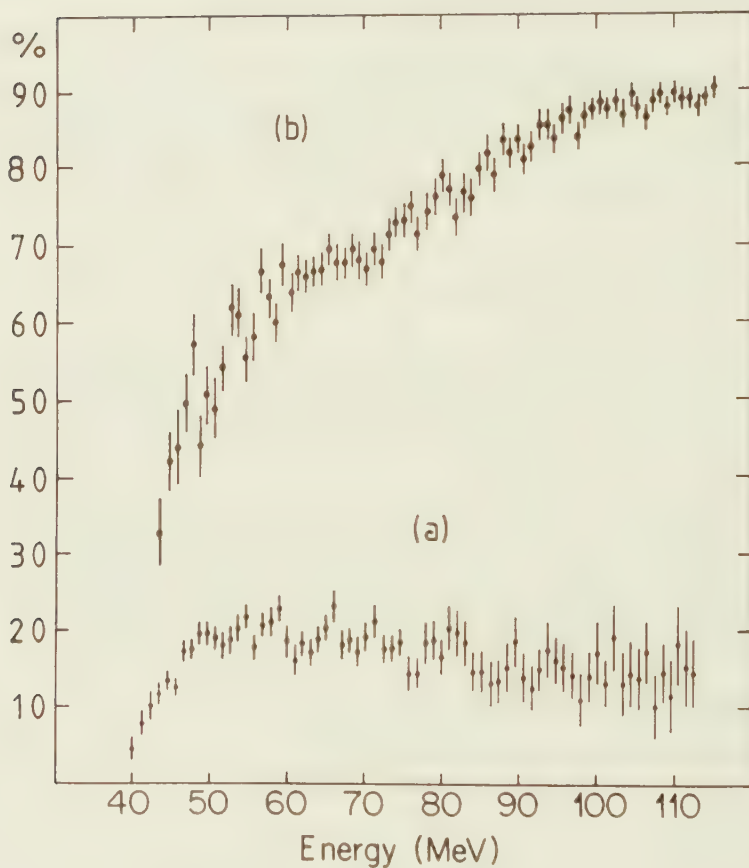


Figure 10.3: Percentage of events that fire three NaI-bars or more. (a) neutrons, (b) photons.

Table 10.2 lists the percentage of neutrons in the final RMC photon spectra for all targets.

10.3 Empty Target Contribution

There exist two types of events that are not associated with the target:

- Photons produced by beam particles stopped in the **degrader**. Their number is strictly proportional to the number of incoming muons. Thus they disappear by subtraction of an empty target run with a subtraction factor:

$$SF = \frac{\# \text{ of incoming muons, target in}}{\# \text{ of incoming muons, target out}}$$

This subtraction is correct, since there was absolutely no material behind the target, i.e. ‘*shadowing effects*’ were excluded.

For the detector D1 the empty target background is rather small ($2 \div 10\%$) while it is $5 \div 25\%$ for D2 (50% for the extraordinarily thin ^{165}Ho target)¹ (see table 10.2).

- **Cosmic ray background:** In principle the photon telescopes of the detectors were protected against charged particles by veto-counters. Unfortunately cosmic muons which crossed the detector D2 under an angle of $\approx 45^\circ$ were able to trigger the telescope by producing Čerenkov light in the lucite light guides. However, these events were scarce and half of them had energies of more than 150 MeV, as shown by measurements with the accelerator switched off.

With the help of these background measurements, it is easy to account for the cosmic events in the RMC-region, extrapolating from the number of events with energies greater than 200 MeV (in this energy region only cosmic events are present).

Since the cosmic background is proportional to the duration of the measurement, it anyway practically vanishes with the empty target subtraction because, for a stable muon intensity, target-in and target-out measurement times become almost equal.

¹Different degrader shielding for each of the two detectors

For D2, about 25% of the total non-target-associated background in the RMC energy region were cosmics (see table 10.2).

10.4 Pion Background

Muon beams are produced by the decay of pion beams. Pion contamination in any muon beam is therefore inevitable. At the SIN superconducting muon channel, this contamination turned out to be $\sim 2 \cdot 10^{-4}$. Since radiative pion capture (RPC) branching ratios are of the order of $1 \div 3\%$ [BCT 77] for all elements (except for hydrogen), the RPC photon yield becomes comparable to that of RMC. Analysis of our data revealed that $\sim 20\%$ of all photons in the RMC region came from RPC for all targets (except for ^{16}O (water target) for which the high hydrogen content doubled the RPC-yield).

However, the pion events can easily be identified by their time structure. While the μ^- -lifetime, even in heavy isotopes as ^{209}Bi , is still ~ 80 nsec, π -capture is always *prompt* in a measurable time scale (strong interaction). Hence the time distribution of muon events in respect to a particle stop signal is exponential whereas pion events are concentrated in a sharp peak (Fig. 5.2).

Fig. 5.3 represents the energy spectrum of prompt and nonprompt events (cuts on time spectrum (Fig. 5.2)). The prompt spectrum clearly shows the enhanced RPC-spectrum which extends up to ~ 140 MeV (the pion mass) while the nonprompt spectrum is nearly empty above 90 MeV. The remaining RPC events are caused by the inefficiency of the beam counter which starts the time measurement and by accidental coincidences of a pion event with a second muon passing through μ_1 just before the pion.

This inefficiency is easily determined when looking at the events between 100 and 140 MeV. In this region only RPC events are present. Thus the inefficiency of prompt-event rejection is:

$$\bar{\epsilon}_{pr} = \frac{N_{np}(100, 140)}{N_{np}(100, 140) + N_{pr}(100, 140)}$$

where N_{np} , N_{pr} denote the sum of nonprompt resp. prompt photon events while the numbers in parenthesis mark the summation limits in MeV.

Since $\bar{\epsilon}_{pr}$ does not depend on the photon energy, one is able to construct a photon spectrum originating only from muons by subtraction:

$$N_{\mu}(E) = N_{np}(E) - S_{pr}N_{pr}(E)$$

where

$$S_{pr} = \frac{\bar{\epsilon}_{pr}}{1 - \bar{\epsilon}_{pr}} = \frac{N_{np}(100, 140)}{N_{pr}(100, 140)}$$

As Fig. 5.2 shows, an appreciable part of the muon induced events fall into the prompt region. We account for them by multiplying $N_{\mu}(E)$ with the 'oversubtraction factor' S_O . Seeing that below 50 MeV [FrT 82], [BCT 77] the RPC spectra drop almost to zero, S_O can readily be found to be:

$$S_O = 1 + (1 + S_{pr})r_{\mu} \quad (10.3)$$

where

$$r_{\mu} = \frac{N_{pr}(< 50)}{N_{\mu}(< 50)}. \quad (10.4)$$

is the fraction of bremsstrahlung events falling into the prompt region.

$\bar{\epsilon}_{pr}$, S_{pr} , r_{μ} and S_O are listed in table 10.1.

10.5 Pile-Up Events

While the detector system D1 had extremely low background from neutrons and cosmic rays, it was much more sensitive to high counting rates in the NaI-crystals caused by the large number of Michel electrons produced in the target. This was due to the fairly big solid angle of the single crystals CV and NaI1 which was $\sim 0,6\%$ for both (for one bar of the NaI wall it was only $0,07\%$) and the fact that the ADC integration times of the D1 crystals ($1,5 \mu\text{sec}$) were 3 times longer than those of the NaI wall. In addition the NaI wall was shielded against low energy electrons by its own lead converter. The isochronous condition (section 11.3) prevented accidental coincidences between several segments of the detector.

DETECTOR 1				
Target	$\bar{\epsilon}_{pr}$	S_{pr}	r_{μ}	S_O
^{12}C				
^{16}O	0.119 ± 0.016	0.135 ± 0.020	0.174 ± 0.003	1.198 ± 0.005
^{27}Al	0.161 ± 0.011	0.192 ± 0.016	0.261 ± 0.003	1.311 ± 0.006
^{40}Ca	0.153 ± 0.010	0.181 ± 0.015	0.158 ± 0.003	1.186 ± 0.004
^{nat}Fe	0.146 ± 0.013	0.171 ± 0.017	0.271 ± 0.005	1.318 ± 0.007
^{165}Ho	0.130 ± 0.018	0.149 ± 0.023	0.214 ± 0.007	1.246 ± 0.009
^{209}Bi	0.184 ± 0.013	0.226 ± 0.020	0.287 ± 0.005	1.325 ± 0.008
Empty	0.351 ± 0.025	0.541 ± 0.058	0.232 ± 0.009	1.358 ± 0.019
DETECTOR 2				
Target	$\bar{\epsilon}_{pr}$	S_{pr}	r_{μ}	S_O
^{12}C	0.112 ± 0.007	0.126 ± 0.009	0.433 ± 0.019	1.488 ± 0.021
^{16}O	0.217 ± 0.007	0.277 ± 0.012	0.182 ± 0.002	1.232 ± 0.004
^{27}Al	0.257 ± 0.008	0.346 ± 0.015	0.205 ± 0.003	1.276 ± 0.005
^{40}Ca	0.231 ± 0.007	0.300 ± 0.011	0.173 ± 0.003	1.225 ± 0.004
^{nat}Fe	0.240 ± 0.007	0.315 ± 0.012	0.209 ± 0.004	1.275 ± 0.005
^{165}Ho	0.325 ± 0.011	0.481 ± 0.025	0.179 ± 0.005	1.265 ± 0.009
^{209}Bi	0.331 ± 0.007	0.496 ± 0.016	0.227 ± 0.003	1.339 ± 0.006
Empty	0.368 ± 0.010	0.582 ± 0.024	0.225 ± 0.006	1.356 ± 0.011

Table 10.1: Quantities involved in the prompt event subtraction

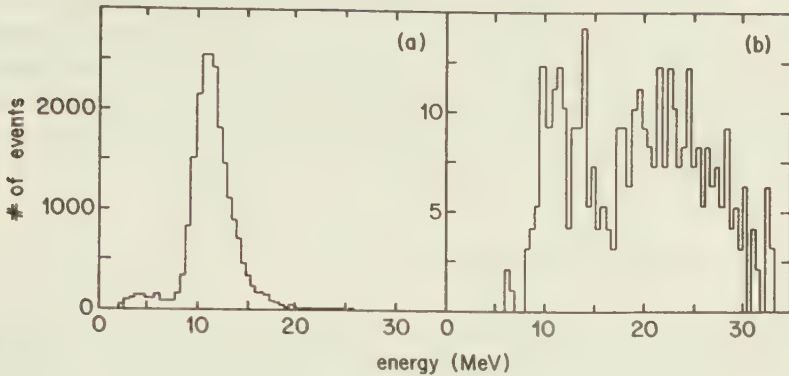


Figure 10.4: NaI converter energy spectrum of Michel electrons from H_2O target. (a):integral spectrum, b): for events with total energy >60 MeV.

10.5.1 Pile-Up Mechanism

For light targets the maximum CV- resp. NaI-counting rates were $5 \div 10$ kHz. From this, an event overlap probability of $\sim 1\%$ was calculated. Therefore the influence of pile-up events was underestimated in a first analysis of the D1 data. This led to RMC yields for ^{12}C and ^{16}O being too large by up to a factor of 3 which unfortunately even were published as preliminary results [Doe+ 86]. The reason for the high photon rates in the RMC energy region was the huge background of bremsstrahlung which was $100 \div 200$ times bigger than the RMC signal. With an additional pulse (caused by a Michel electron) falling into the $1.5 \mu\text{sec}$ NaI ADC gate, these low energy background events were shifted into the RMC energy region and finally gave a contribution of the same order of magnitude as the real RMC events. By means of a Monte Carlo simulation it has been shown that the energy spectrum of pile-up events is very similar to the expected RMC spectrum and therefore it does not look conspicuous at all.

Pile-up events were first discovered in the measured Michel electron spectra of light targets, where $\sim 1\%$ of strange high energy events (>60 MeV) appeared. They can easily be identified as pile-up events looking at their CV energy spectrum (Fig. 10.4).

The analysis of the D2 electron spectra proved that no pile-up was present there. For this test, electron spectra with low event rate (closed intensity slits, $\sim 350\text{kHz}$ muon beam) were compared to high rate measurements ($\sim 25\text{ MHz}$ muon beam): neither appeared high energy events above the Michel spectrum nor did the number of isochronously hit bars per event change.

10.5.2 Pile-Up Suppression

It has been tried to suppress pile-up events by accepting only D1 photon events which have energies of less than 11.5 MeV in CV (pile-up is always bound to happen simultaneously in CV and NaI1, since electrons have to pass through CV to get into NaI1).

The reduction of the detector acceptance by this cut has been determined with RPC photons. The RMC photon yield for the ^{16}O target has indeed been reduced by a factor of ~ 2 .

Nevertheless, the efficiency of this condition is very uncertain, since photons that convert only in the rear part of CV can deposit very little energy in this counter so that even with some additional energy from a pile-up electron they do not exceed the 11.5 MeV limit.

10.5.3 Pile-Up Simulation

To have a fairly reliable correction of the influence of pile-up on the D1 RMC-spectra for medium heavy targets (^{27}Al , ^{40}Ca) the above described pile-up process has been simulated by a Monte Carlo program.

The parameters for this calculation are all known:

- For the *bremsstrahlung energy spectrum*, the total measured photon spectrum from the target was used. Since only $\sim 1\%$ of all events are RMC photons, this is a very good approximation.
- For the *Michel electron spectrum* the theoretical spectrum from [HVR 74] was used.

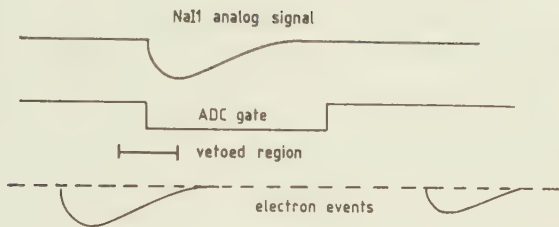


Figure 10.5: Pile-up simulation. Basic principle.

- The *electron rate* was measured in the experiment as the rate of $\check{C}2 \wedge T2$ coincidences (corrected by the T2 and $\check{C}2$ efficiencies).
- The *NaI pulse shape* is known from an earlier measurement by our group [DD 82].
- The *ADC gatelength* is known from the experiment as well as the length of the anticounter veto which, in principle, would have permitted to exclude pile-up by electrons (but obviously was too short).

The simulated situation is depicted in Fig. 10.5.

The upper part of the picture shows the bremsstrahlung input pulse (energy spectrum is the measured one). From the Poisson-distribution, which is given by the measured electron frequency, the positions of the last and the next electron pulse are drawn (higher order coincidences are completely negligible). Their energy spectrum is the theoretical one, corrected by the energy loss in the target. Pulses which fall into the 300 nsec anticounter veto are disregarded. The final energy of the event is then given by the primary photon energy plus the part of the electron energy which falls into the ADC gate.

The program has been tested with the Michel spectrum for the H_2O target (Fig. 10.6). For this purpose the calculated Michel spectrum was taken as input spectrum. The result was quite satisfactory.

The *correction factors* for the RMC-yield calculated by this procedure are listed in table 10.2. With this correction the photon yields measured with D1 become consistent with those of D2.

However, for ^{16}O , the correction is so huge that the yield measured with

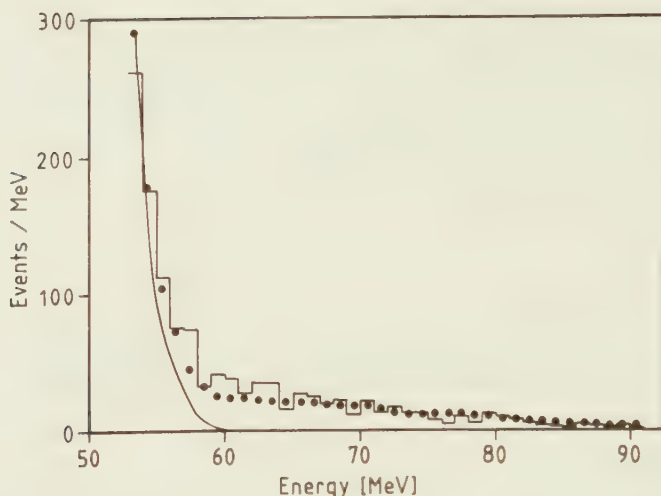


Figure 10.6: Pile-up simulation for electrons from the water target. Solid line: theoretical Michel spectrum folded with exp. response function. Histogram: measured energy spectrum. Points: simulated spectrum.

D1 cannot be regarded as a serious result, but rather as plausibility check for the reliable results of D2. The ^{12}C -data of D1 have been discarded, since they consisted almost completely of pile-up events.

10.6 Construction of Background-free Spectra

The following procedure of sequential background subtraction leads to 'clean' RMC spectra and to the correct total number of RMC events in an energy region:

- *Prompt-subtraction*: RPC-subtraction is done separately for target-in and target-out.

$$\text{target in} : N_{\mu}^{\text{in}}(E) = N_{np}^{\text{in}}(E) - S_{pr}^{\text{in}} N_{pr}^{\text{in}}(E)$$

$$\text{target out} : N_{\mu}^{\text{out}}(E) = N_{np}^{\text{out}}(E) - S_{pr}^{\text{out}} N_{pr}^{\text{out}}(E)$$

The prompt subtraction has to be done before the target-out subtraction because the fraction of prompt events (S_{pr}^{in} and S_{pr}^{out}) are different ²(see table 10.1).

- The subtraction of *non target-associated background* is performed in two stages. First *target out* data is subtracted:

$$N'_\mu = N_\mu^{in}(E) - \frac{S_O^{out}}{S_O^{in}} S_F N_\mu^{out}(E) \quad (10.5)$$

S_O^{out}/S_O^{in} accounts for the different oversubtraction factors.

From $N'_\mu(E)$ the factor for the *cosmics subtraction* can be determined:

$$S_{cos} = \frac{N'_\mu(> 200)}{N_{cos}(> 200)} \quad (10.6)$$

where $N_{cos}(E)$ is the cosmic spectrum measured with the accelerator stopped.

Thus, the 'clean' spectrum from target is obtained as:

$$N_{tg}(E) = N'_\mu(E) - S_{cos} N_{cos}(E) \quad (10.7)$$

- For $N_{tg}(E)$ the *neutron content* $N_n(E)$ (resp. the overall fraction f_n) and the fraction f_{pu} of *pile-up events* are determined (equation 10.2). Accounting in addition for the *oversubtraction* the final photon spectrum is as follows:

$$N_\gamma(E) = S_O^{in}(1 - f_n)(1 - f_{pu})N_{tg}(E) \quad (10.8)$$

This assumes that the neutron and pile-up contaminations are small and their shape is similar to the RMC spectrum (see table 10.2).

Remark:

A correct determination of S_{pr}^{in} , S_{pr}^{out} and S_O^{in} , S_O^{out} requires the cosmic subtraction to be done first on the individual nonprompt and prompt spectra,

²This has geometrical reasons: The time giving beam counter has a non uniform efficiency. Pions entering via the center of the counter are likely to stop in the degrader ($\rightarrow$ target-out signal) while those coming in at the periphery mainly stop in the target.

Target	f_n (Neutrons)	Empty Target Contribution		f_{pu} (Pile-up)	Cosmic Events
		D1	D2		
^{12}C	-	-	$.522 \pm .98$	-	$-.68 \pm .1$
^{16}O	$.099 \pm .020$	$.065 \pm .016$	$.182 \pm .018$	.632	$.083 \pm .005$
^{27}Al	$.070 \pm .009$	$.019 \pm .005$	$.086 \pm .009$	.258	$.012 \pm .001$
^{40}Ca	$.053 \pm .006$	$.014 \pm .003$	$.049 \pm .005$	.022	$.020 \pm .001$
^{nat}Fe	$.049 \pm .006$	$.020 \pm .005$	$.060 \pm .006$	.025	$.011 \pm .001$
^{185}Ho	$.116 \pm .048$	$.080 \pm .019$	$.345 \pm .034$	.009	$.259 \pm .018$
^{209}Bi	$.185 \pm .019$	$.064 \pm .016$	$.201 \pm .021$	.015	$.027 \pm .002$

Table 10.2: Background contributions in the RMC' region. The empty target contribution is defined as $(S_O^{out} N_\mu^{out}) / (S_O^m SF N_\mu^m)$, the fraction of cosmic events is $S_{cos} N_{cos} / N_\mu'$

but - these factors once calculated - the subtraction can be carried out exactly as described above.

Chapter 11

Particulars on Detector Features

11.1 Energy Calibration of D1

11.1.1 NaI Converter (CV)

The CV energy scale has first been fixed by setting the ‘pedestal’ of its ADC to zero energy. The *pedestal* (or *noise peak*) of an ADC can be produced by reading it at random times without any signal from the detector.

Michel electrons passing through the whole CV counter provide a peak at ~ 11 MeV (determined with EGS Monte Carlo program). The position of this peak has been taken as a second reference point to calculate the linear energy scale (Fig. 11.1). Daily repetitions of electron runs proved the perfect stability of the counter.

11.1.2 NaI1

To calibrate the big NaI1 crystal the photon spectrum of RPC on hydrogen (a monoenergetic peak at 130 MeV ($p\pi^- \rightarrow n\gamma$) and a continuous box spectrum from 55 to 83 MeV ($\pi^0 \rightarrow 2\gamma$)) and again the ADC pedestal have been used. The hydrogen spectrum has been produced by subtraction of the spectra from a carbon and a CH_2 -target. Only ‘nonconverted’ photons have been accepted, i.e. events that only convert in the NaI1, but give no signal in the telescope counters. The theoretical spectrum has been folded with a phenomenological resolution function and fitted to the experimental spectrum with the energy calibration factor as a free parameter (Fig. 11.2).

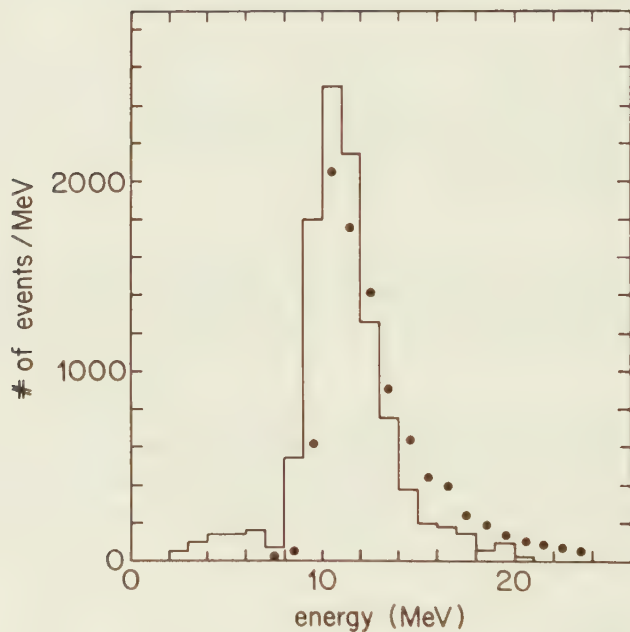


Figure 11.1: Energy spectrum of Michel electrons crossing the NaI converter. The points represent a simulation.

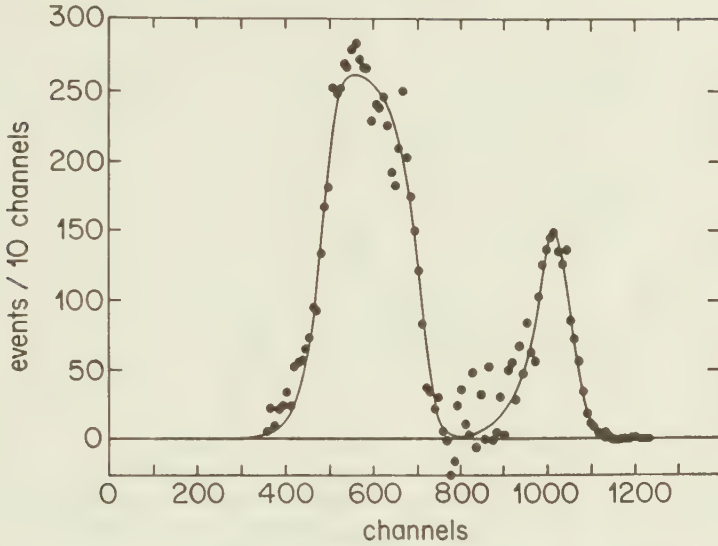


Figure 11.2: RPC on hydrogen. Energy spectrum of nonconverted photons in NaI1. The solid line is the fit which determines the energy calibration factor.

The resolution function $R_1(E_o, E)$ is a gaussian with a low energy tail of the following form:

$$R_1(E_o, E) = A e^{-\frac{1}{2\sigma^2}(E-E_o)^2} \quad \text{for } E > E_t \quad (11.1)$$

$$R_1(E_o, E) = A e^{-\frac{1}{2\sigma^2}(E-E_o)^2} \cdot e^{\frac{1}{2\sigma^2}(E-E_t)^2} \quad \text{for } E < E_t$$

where $E_t = E_o - 0.75853 \sigma$. A is numerically determined by the normalization condition for $R_1(E_o, E)$:

$$\int_{-\infty}^{\infty} R(E_o, E) dE = 1$$

σ has been determined from $\Delta E/E(FWHM) = 2.35\sigma/E$ at 130 MeV (which was 9%) with the assumption that $\Delta E/E$ is constant for all energies.

The energy calibration of NaI1 has been monitored with the spectrum of cosmic muons crossing the detector (cosmic veto counter in coincidence). See Fig. 11.3.

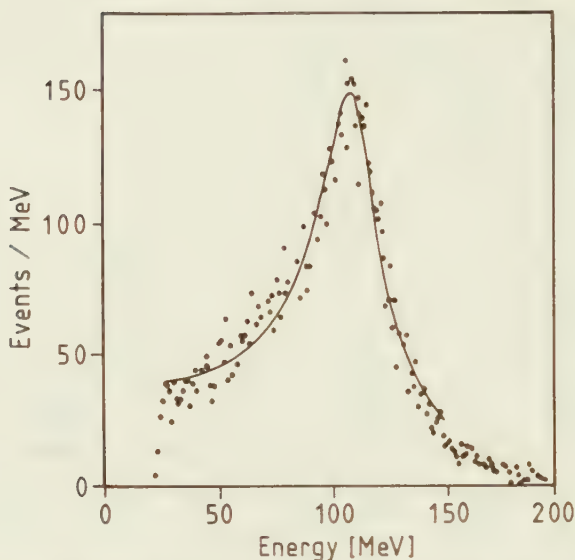


Figure 11.3: Energy spectrum of cosmic muons passing through NaI1.

During 8 days of operation a shift of 2% has been observed.

11.1.3 The Total Photon Energy

The total energy of a converted photon equals the sum of the C_V- and NaI1- energy plus a correction for the energy loss in the other telescope counters C₁ and T₁ and in the sealed aluminium housings of the NaI crystals.

The mean energy loss calculated for a minimum ionizing electron-positron pair is 5.5 MeV. The energy sum spectrum was fitted again with the hydrogen RPC-spectrum (folded with the same line shape (11.1) with different resolution) in order to check the calibration (figure 4.1).

Two facts were apparent from this check:

- 1) The mean missing energy at 60 MeV was ~ 7.5 MeV (~ 2 MeV larger than expected).
- 2) The energy scale was slightly nonlinear, i.e. the mean energy loss

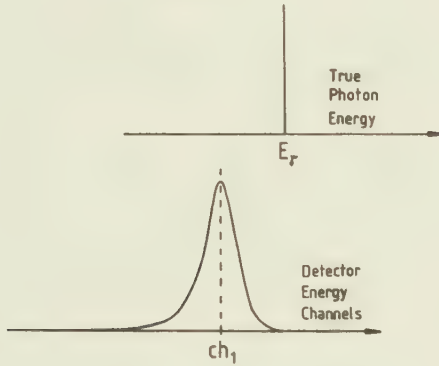


Figure 11.4: Meaning of energy calibration.

depends on the photon energy. (Possible reason: the energy spectrum of the thin trigger counter showed that the number of electrons leaving the converter clearly increases with the photon energy. This was confirmed by a Monte Carlo simulation (c.f. next section)).

Thus, the final energy calibration for the detector D1 has been established as follows:

$$E_{\gamma} = \frac{ch_1 + \omega_1}{1 - \epsilon_1(ch_1 + \omega_1)} \text{MeV} \quad (11.2)$$

$$\omega_1 = 1.4 \pm 0.8, \quad \epsilon_1 = 2.87 * 10^{-4}$$

where ch_1 is the channel of the energy sum spectrum, ω_1 the fitted offset and ϵ_1 the fitted nonlinearity.

The exact meaning of energy calibration is detailed by Fig. 11.4 : The channel ch_1 in a measured energy histogram represents the center of the response $R(E_o, E)$ to a monoenergetic photon line of true energy E_{γ} (equation 11.1).

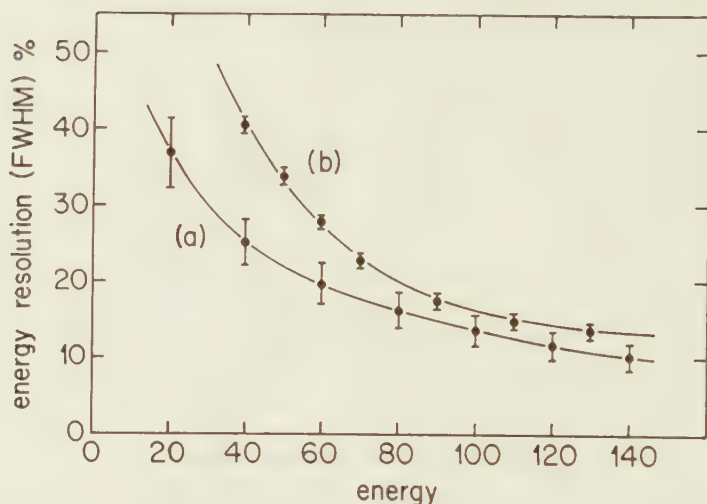


Figure 11.5: Resolution $\Delta E/E(FWHM)$ versus energy E_γ for the two detector systems: (a) D1, (b) D2.

11.2 Acceptance of D1

11.2.1 Resolution Function

The lineshape of D1 was assumed to match the form of equation 11.1, i.e. the same as for NaI1 alone, but now with an *energy dependent relative resolution*:

$$\frac{\Delta E}{E_\gamma}(FWHM) = \sqrt{\alpha_1 + \frac{\beta_1}{E_\gamma}} \quad (11.3)$$

α_1 and β_1 were determined by a χ^2 -fit to the hydrogen RPC spectrum (Fig. 4.1):

$$\alpha_1 = -0.011, \quad \beta_1 = 3.0$$

Equation 11.3 is visualized in Fig. 11.5.

11.2.2 Relative Acceptance

The energy dependence of the detector acceptance $r_1(E)$ was theoretically calculated using the photon conversion total cross section of sodium iodide [HUB 80].

Low energy photons deposit a large fraction of the total energy in the conversion telescope, already. For a photon converting in the first part of the converter and then undergoing a rapid shower development, this energy can amount to 40 MeV. Since the energy threshold of NaI1 was set to 22 MeV, such a photon is eventually no longer able to trigger the whole system. The influence of this effect on the detector acceptance has been measured using RPC photons from a CH_2 target.

For energy slices of 5 MeV width, the number of accepted photons with a 22 MeV threshold has been compared to the one with a very low energy threshold.

The relative acceptance $r_1(E)$ is normalized to 1 at 130 MeV.

11.2.3 Absolute Acceptance

The absolute value of the detector acceptance has been established with a carbon RPC measurement. From the measured photon spectrum $N_\gamma(E_i)$ the absolute acceptance at 130 MeV is derived as:

$$a_1(130\text{MeV}) = \frac{\sum_i N_\gamma(E_i) r_1(E_i)}{f_{cap} B_\gamma^c (1 - f_{abs}) \pi_{st} (1 - f_{veto})} \quad (11.4)$$

where

$r_1(E)$ is the relative acceptance as calculated above, normalized to 1 at 130 MeV.

f_{cap} is the fraction of pions captured at a ^{12}C nucleus [Per 79].

B_γ^c is the ^{12}C -RPC branching ratio [Per 79].

f_{abs} is the fraction of photons absorbed in the target (calculated with Monte Carlo simulation).

π_{st} is the number of pions stopped in the target.

f_{veto} is the fraction of the total measuring time during which the detector was dead because of an A1- or cosmic veto.

$$\begin{aligned}
 \sum_i N_\gamma(E_i) r_1(E_i) &= (3254 \pm 70) \\
 f_{cap} B_\gamma^c &= (1.770 \pm 0.054) 10^{-2} \\
 f_{abs} &= 0.019 \\
 \pi_{st} &= (164 \pm 12) 10^6 \\
 f_{veto} &= 0.034
 \end{aligned}$$

and finally

$$\underline{a_1(130MeV) = (1.18 \pm 0.09)10^{-3}}$$

The final expression for the detector acceptance reads then:

$$a_1(E) = a_1(130MeV)r_1(E)$$

It is displayed in Fig. 4.3.

11.3 NaI Wall Specials

A complete description of the NaI wall and the appertaining electronics is given in [Bay 86].

11.3.1 Operation

In short terms, the 8 x 8 NaI matrix is operated as follows:

The photomultiplier signals of the individual bars are actively splitted into three parts:

- The first signal goes to a common ‘trigger box’ where the total energy of any group of 4 adjacent bars is discriminated.
If the energy in one of the 49 possible groups exceeds a selected threshold a trigger signal ‘NaIH’ (NaI high) is released.

- The second signal goes into an individual constant fraction discriminator (CFT) with a threshold of ~ 1 MeV. The CFT outputs of the inner 6×6 bars are synchronized with individual delays and go into a 36 fold logic OR called NaIL (NaI low). An event in the NaI wall is then given by the logical AND of $\text{NaIL} \wedge \text{NaIH}$, where NaIL determines the time. This signal can be used for global timing purposes.

Another CFT output signal starts an individual TDC (called TOF, time of flight) which is then stopped by the next synchronisation RF (Repetition Frequency) pulse of the accelerator.

- The third analog signal is fed to an individual ADC providing the energy information.

11.3.2 Event Processing

For each event, the bar with the maximum energy is determined first. Then one selects all bars having a TOF which lies within a time interval of $\pm \Delta t$ (10 nsec in our case) around the TOF of the bar with maximum energy (isochronous condition) thus suppressing pile-up effects. The total energy and the centroid of the energy deposit are calculated from the isochronously hit bars. Events with a centroid lying outside the inner 6×6 square are discarded (in order to reduce the probability of shower escape).

11.4 Monte Carlo Simulation of D2

To establish a reliable estimate of the response function of D2, the detector has been simulated by the EGS ¹ Monte Carlo program developed at SLAC. Fig. 11.6 shows an example for a simulated event.

Photons were generated uniformly on the surface of the lead converter heading in radial direction away from the target. Basically the program calculated the energy deposit in every counter. In order to account for the finite energy resolution of a NaI module, [Bay 86], a random-distributed

¹Electron Gamma Shower

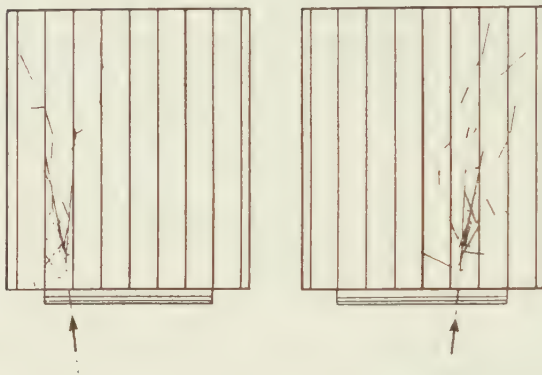


Figure 11.6: Top and side view of a simulated 100 MeV photon event.

noise was added to the energy of each hit bar. The event was then processed under the same conditions as the real ones.

In steps of 10 MeV about 30.000 events with energies $E_\gamma = 10, 20, \dots, 140$ MeV were generated. Each resulting distribution of total energy was fitted with a resolution function $R_2(E_0, E)$ which is rather similar to $R_1(E_0, E)$ (see equation 11.1), but has a variable slope of the low energy tail:

$$R_2(E_0, E) = Ae^{-\frac{1}{2\sigma_1^2}(E-E_0)^2} \quad \text{for } E > E_t \quad (11.5)$$

$$R_2(E_0, E) = Be^{-\frac{1}{2\sigma_2^2}(E_t - E)} \quad \text{for } E < E_t$$

with

$$E_t = E_0 - \frac{\sigma_1^2}{2\sigma_2}$$

and

$$B = Ae^{-\frac{(E_t - E_0)^2}{\sigma_1^2}}$$

where σ_2 is a free parameter. For an example see figure 11.7.

From the fit of $R_2(E_0, E)$ to the simulated peaks follows the mean missing energy $(E_\gamma - E_0)$ in function of the true photon energy E_γ as well as σ_1 , σ_2 and the relative acceptance $r_2(E)$ (Fig. 11.8).

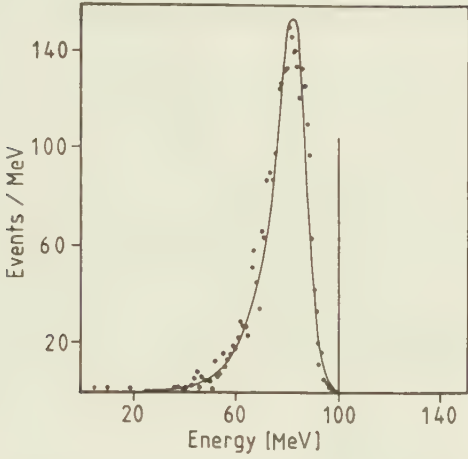


Figure 11.7: EGS generated response of the NaI wall to 100 MeV photons.

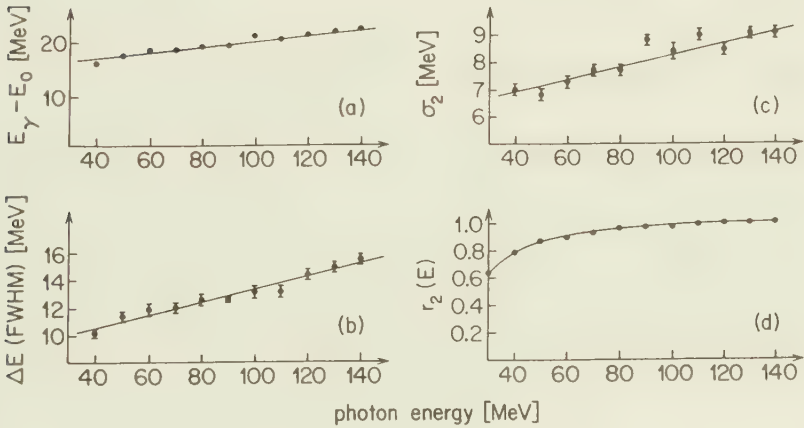


Figure 11.8: Results of the Monte Carlo simulation. The mean energy loss $E_\gamma - E_0$ (a), the resolution (FWHM) (b), the slope σ_2 of the low energy tail (c) and the relative acceptance (d) are displayed.

$E_\gamma - E_0$, σ_1 and σ_2 are then parametrized as follows:

$$\begin{aligned} E_\gamma - E_0 &= \omega_2 + \epsilon_2 E_\gamma \\ \sigma_1 &= \alpha_2 + \beta_2 E_\gamma \\ \sigma_2 &= \varphi + \delta E_\gamma \end{aligned} \tag{11.6}$$

The solid lines in Fig. 11.8 represent the following parametrization:

$$\begin{aligned} \omega_2 &= 15.0 \text{ MeV} \\ \epsilon_2 &= 0.051 \\ \alpha_2 &= 3.7 \text{ MeV} \\ \beta_2 &= 0.020 \\ \varphi &= 6.1 \text{ MeV} \\ \delta &= 0.022 \end{aligned}$$

The relative acceptance $r_2(E)$ (normalized to 1 at 130 MeV) is taken as an interpolated polygone through the values resulting from the simulation.

11.5 Energy Calibration of D2

To check the Monte Carlo simulation and confirm the energy calibration, the hydrogen RPC spectrum was folded with the above obtained resolution function and fitted to the experimental data (Fig. 4.2) with ω_2 as a free parameter. The fit gave a best value of (18.1 ± 0.7) MeV for ω_2 instead of 15.0 MeV as claimed by the simulation. This was due to a 1mm steel coating having been missinterpreted as aluminium. Thus, the final energy calibration of D2 is given by:

$$E_\gamma = \frac{ch_2 + \omega_2}{1 - \epsilon_2} \tag{11.7}$$

where ch_2 is the histogram channel of the total energy in D2, $\omega_2 = 18.1$ MeV and $\epsilon_2 = 0.051$.

11.6 Absolute Acceptance of D2

The acceptance of D2 as a function of energy is written as:

$$a_2(E_\gamma) = a_2(130\text{MeV})r_2(E_\gamma)$$

in analogy to the D1 acceptance.

Yet, for D2, $a_2(130\text{ MeV})$ was determined with the monoenergetic peak of the RPC branching ratio on hydrogen, since the low energy tail of the carbon spectrum was cut off by the NaI2 energy threshold. According to equation 11.4 we may write:

$$a_2(130\text{MeV}) = \frac{N_\gamma^{RPC}}{f_{cap}^H B_\gamma^{RPC} (1 - f_{abs}) \pi_{st} (1 - f_{veto})}$$

where

- f_{cap}^H is the fraction of pions captured by a proton in the CH_2 compound [Mor 79].
- B_γ^{RPC} is the branching ratio for the reaction $(\pi^- p \rightarrow n \gamma)$ (Panofsky ratio [Coc+ 61], [Rya 63]).
- N_γ^{RPC} is the number of photon events in the 130 MeV peak (c.f. Fig. 4.2).
- The remaining quantities have been explained for equation 11.4.

With

$$\begin{aligned} N_\gamma^{RPC} &= (6884 \pm 100) \\ f_{cap}^H &= (0.0126 \pm 0.0006) \\ B_\gamma^{RPC} &= (0.395 \pm 0.003) \\ f_{abs} &= 0.019 \\ \pi_{st} &= (374 \pm 26) 10^6 \\ F_{veto} &= 0.007 \end{aligned}$$

one gets then

$$a_2(130\text{MeV}) = (3.80 \pm 0.33)10^{-3}.$$

Confirmation : From two pion runs - with a ,unfortunately, unknown number of π -stops - the ratio of the acceptances of the two detectors has been determined independently. It agrees to 1.5% (with an error of 4%) with the ratio of the above established acceptances.

For RMC' runs only events with at least 3 hit bars were accepted in order to suppress neutron events. The reduction of the photon acceptance by this condition was measured with RPC' photons from a CH_2 target (figure 10.3). The reduced photon acceptance was fitted using a function of the form

$$1.03 - 0.898e^{-0.0222ch_2}$$

with which $a_2(E)$ has been multiplied for the analysis of RMC' runs. The resulting acceptance is displayed in figure 4.3. (b).

Chapter 12

Particle Beams

12.1 Muon Beam

12.1.1 Beam Structure

Fig. 12.1 shows the time structure of the muons stopped in the ^{40}Ca target. The contamination with pions ($\sim 10^{-4}$) and electrons ($< 1\%$) is very small and not visible, therefore. It has been neglected for the calculation of stop rates.

Fig. 12.2 represents the muon range curve for the ^{40}Ca target.

12.1.2 Counter Efficiencies

The efficiencies of μ_2 and μ_3 were found to be 99.1% and 94.4% respectively by sandwiching them between two other scintillation counters. The reasons for the low efficiency of μ_3 are its modest thickness (1 mm) and the rather complicated light guide.

For measurements with high muon rates, the stop rates were normalized to the μ_1 rate (all other counters were switched off). Therefore, the rate dependence of this counter hodoscope was studied closely. Fig. 12.3 displays the μ_1 -counting rate versus the rate of the proton beam monitor. The fitted curve is of the following form:

$$\nu' = \nu \frac{R}{1 + \tau \nu} \quad (12.1)$$

where ν' is the measured μ_1 -rate, ν the true μ_1 -rate and νR the mon-

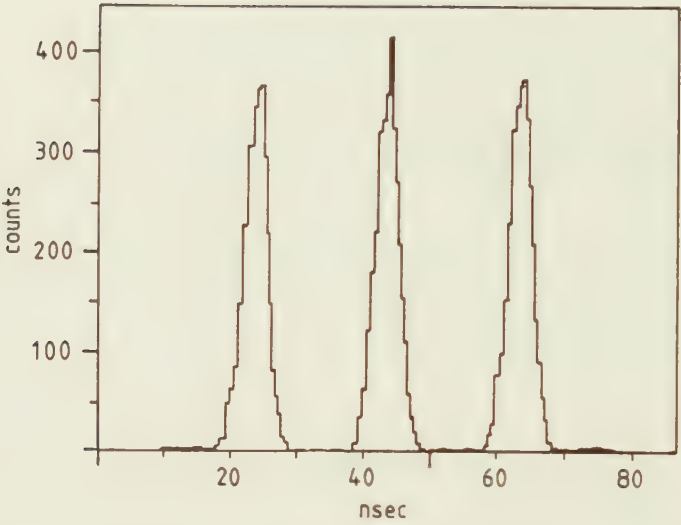


Figure 12.1: Time structure of stopped muons.

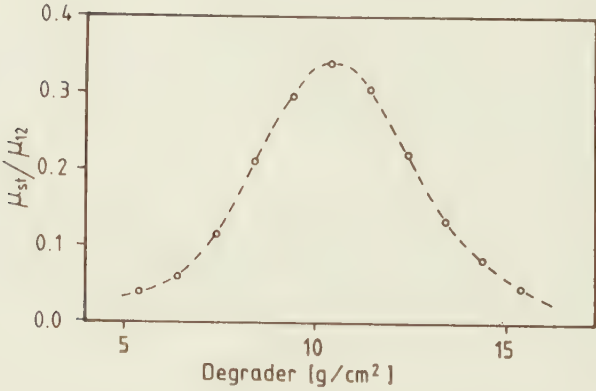


Figure 12.2: Muon rangecurve for the 5.0 g/cm² ⁴⁰Ca target.

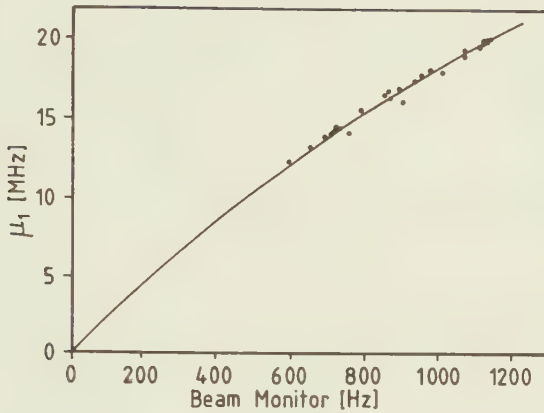


Figure 12.3: Determination of the hodoscope deadtime correction.

itor rate. τ is the deadtime correction. The curve fits quite well for a hodoscope deadtime of 14 nsec.

A further correction to the μ_1 -rate was made regarding its noise rate which was 6 kHz with the accelerator off.

The inefficiency of μ_4 can be found out from the empty target range curve. Since no particles stop in the target, a stop signal comes either from a particle stop in μ_3 or is caused by the inefficiency of μ_4 . Hence, one may write:

$$\mu_{st} - \mu_{st}(\mu_3) = (\mu_{in} - \mu_{st}(\mu_3))\bar{\epsilon}_{\mu_4}$$

where $\mu_{st}(\mu_3)$ is the stop rate of muons in the μ_3 scintillator, μ_{in} stands for $\mu_1 \wedge \mu_2 \wedge \mu_3$ and $\bar{\epsilon}_{\mu_4}$ is the μ_4 inefficiency. $\mu_{st}(\mu_3)$ can be calculated from the μ_{in} -range curve. Accordingly, $\bar{\epsilon}_{\mu_4}$ can be regarded as the gradient of a plot of $(\mu_{st} - \mu_{st}(\mu_3))$ versus $(\mu_{in} - \mu_{st}(\mu_3))$ as displayed in figure 12.4.

The nonlinearity of this plot for small thickness of the degrader (right hand side) is caused by the low energy loss of the unmoderated muons in μ_4 (increase of inefficiency). For the same reason, μ_4 was almost completely inefficient for electrons.

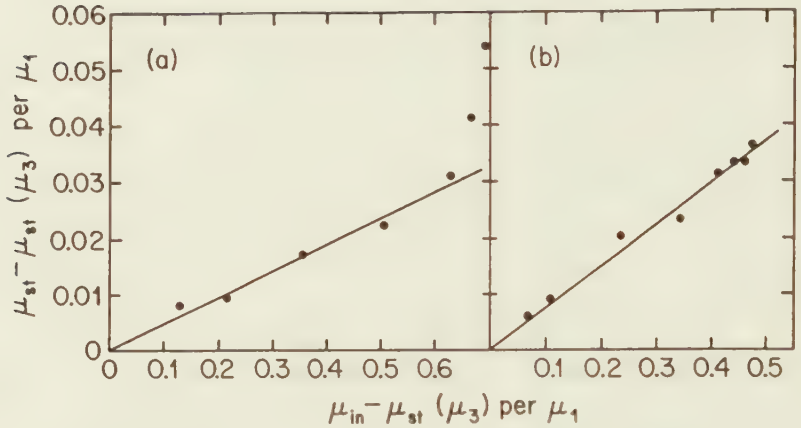


Figure 12.4: Determination of μ_4 -inefficiency for the muon (a) and pion (b) beam. The straight line represents an inefficiency of 5%, resp. of 7.5%.

12.1.3 Muon Stop Rates

The muon stop rates have been determined for all targets at low beam intensity (350 kHz, slits closed) using the formula:

$$\mu_{st}^{true} = \frac{\mu_{st} - \mu_{st}(\mu_3) - (\mu_{in} - \mu_{st}(\mu_3))\bar{\epsilon}_{\mu_4}}{\epsilon_{\mu_4}} \frac{1}{\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}} \quad (12.2)$$

The true μ_1 -rate, μ_1^{true} was calculated with the deadtime correction (equation 12.1).

The calculation of the stop rates with high beam intensity is then straight forward:

$$\mu_{st}^{true}(h.i.) = \frac{\mu_{st}^{true}}{\mu_1^{true}}(l.i.) \mu_1^{true}(h.i.) f_{slits} \quad (12.3)$$

where ‘h.i.’ and ‘l.i.’ mean ‘high intensity’ and ‘low intensity’, respectively.

f_{slits} is a rather important correction factor of 1.48 ± 0.04 which can be explained as follows:

the beam foci at the positions of the intensity slits and the target (figure 3.1)

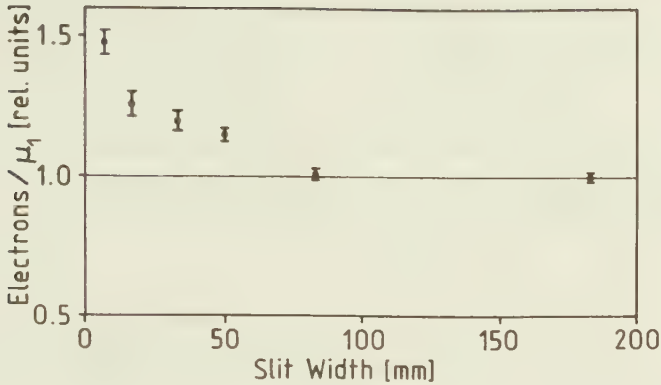


Figure 12.5: Electron signal (D2) from ^{40}Ca target normalized per μ_1 versus the width of the intensity slit.

are far from ideal. Therefore, the closed slits cut out a large halo of the beam. With open slits this halo passes then through μ_1 , but fails to be projected into the target focus. Since the cross sections of all targets are practically identical this effect is the same for all targets. It was determined for all slit positions by means of the Michel electron rate from the target (which is proportional to the muon stop rate). Figure 12.5 shows the electron signal from the ^{40}Ca target for various slit widths. The stop rate measurements were made with a 7 mm opening of the slits, while they were completely open (183 mm) during the RMC runs.

12.2 Pion Beam

12.2.1 Beam Structure

Figure 12.6 shows the structure of the pion beam within one repetition period of the accelerator. Since pions are extracted from the muon channel at the same momentum as they are injected, the composition of the beam is dominated by electrons. A rather broad background of muons is also distinguishable between the electron and the pion peak.

In order to obtain a clean pion beam, only particle stops which fell into the ‘pion-window’ indicated in figure 12.6 were counted, and only events

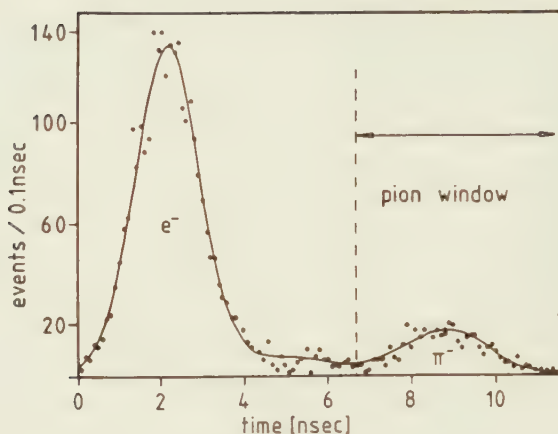


Figure 12.6: Time structure of the pion beam.

immediately following such a stop were accepted and recorded.

12.2.2 Pion Stop Rates

Since each event requires a pion stop, the beam particles which are not detected and thus the inefficiencies of π_1, π_2, π_3 are of no importance for the absolute pion stop rate (n.b. the names of $\mu_1, \mu_2, \mu_3 \dots$ will be replaced by $\pi_1, \pi_2, \pi_3 \dots$ in the following). In contrast to that, falsely counted particle stops become significant. There are three mechanisms contributing to that:

1. As illustrated by figure 12.6, a part of the flat **muon contamination** falls into the pion time window. This contribution has been estimated at both ends of the pion window by fitting gaussians to all peaks in the time spectrum. This yielded a result for the contamination of $f_{cont} = 0.098 \pm 0.030$.
2. The **inefficiency** of π_4 leads directly to false pion stops. It was determined in the same way as for muons (figure 12.4).
3. The relatively large coincidence time of $\mu_1 \wedge \mu_2 \wedge \mu_3$ in the μ_{in} -electronic allowed **accidental coincidences** of pions, stopped in the degrader, with electrons flying through the whole beam counter

telescope. Since π_4 was almost completely inefficient for electrons, a π_{st} -signal was produced. As π_1 defined the time of the π_{st} -coincidence, these events fell into the pion window. This accidental stop rate is called ' π_{st}^{acc} '.

With all particle rates and coincidence gate widths being known, it is easy to calculate the appropriate correction to the pion stop rate. $(3 \pm 1)\%$ of all pions stopped in the degrader got a false target stop signal.

Hence the number of true pion stops is calculated in analogy to 12.2:

$$\pi_{st}^{true} = \frac{\pi_{st} - \pi_{st}^{acc} - \pi_{st}(\pi_3) - \bar{\epsilon}_{\pi_4}(\pi_{in} - \pi_{st}(\pi_3) - \pi_{st}^{acc})}{\epsilon_{\pi_4}} (1 - f_{cont}) \quad (12.4)$$

Chapter 13

Details on Yield Calculation

The formula by which the final RMC branching ratios are calculated is given in section 6. It is repeated here together with additional information on some quantities involved:

$$N_{tg} = B_{RMC}(\Delta E) \int_{-\infty}^{\infty} r(E) T_N(E) r(E, ch) dE$$
$$\cdot \frac{\mu_{st} f_{cap} (1 - f_{abs}) (1 - f_{veto}) a(130 \text{ MeV})}{S_0 (1 - f_n) (1 - f_{pu})} \quad (13.1)$$

13.1 Folding and Unfolding

In a first step the theoretical spectrum is corrected to the energy-dependent acceptance of the detector. The folding procedure is graphically represented in figure 11.4.

The concept of interpreting the center of the resolution function as the energy (in channels) corresponding to the monoenergetic line, consequently leads to unphysically high energies appearing in the energy spectra. (Example: the folded bremsstrahlung spectrum in figure 5.1 extends up to channel 62, hence to about 64 MeV, while the unfolded spectrum vanishes at ~ 57 MeV). Yet, consistently applied, this means just a shift of the energy scale.

This raises the question if the experimental spectra could be *unfolded* just by shifting them into the correct energy scale and by dividing them by the detector acceptance. [Fri + 85] showed that this is feasible for featureless spectra and high energy resolution (2 MeV). Figure 13.1 displays the Primakoff polynomial in its original shape (a) and folded with

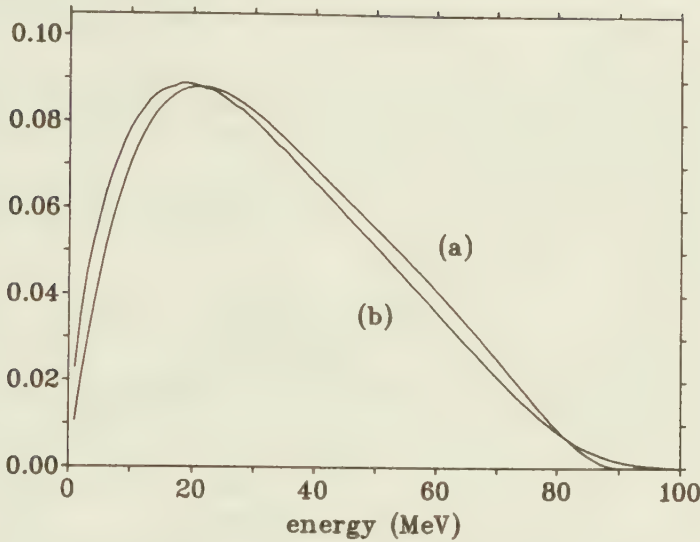


Figure 13.1: Theoretical RMC yield (a) folded with the resolution function of D1 and unfolded again by the method described in the text (b).

the resolution function of D1 and unfolded again by the above mentioned method (b), proving that the distortion of the spectrum by the detector resolution is considerable at the end of the spectrum. At 60 MeV it is still nearly 15%. Thus this unfolding procedure is not perfectly suited for the present case. Figure 13.2 shows examples of unfolded spectra for ^{209}Bi and ^{nat}Fe . The yields are normalized per captured muon by multiplying with a factor

$$C_{norm} = \frac{S_O(1 - f_{pu})(1 - f_n)}{\mu_{st} f_{cap} a(130\text{MeV})(1 - f_{abs})(1 - f_{veto})}$$

A second possibility of unfolding RMC spectra was pointed out by [Har+ 77] who found a numerical solution to the unknown $T(E)$ in equation 13.1. This method is hopeless for spectra of poor statistics as presented here.

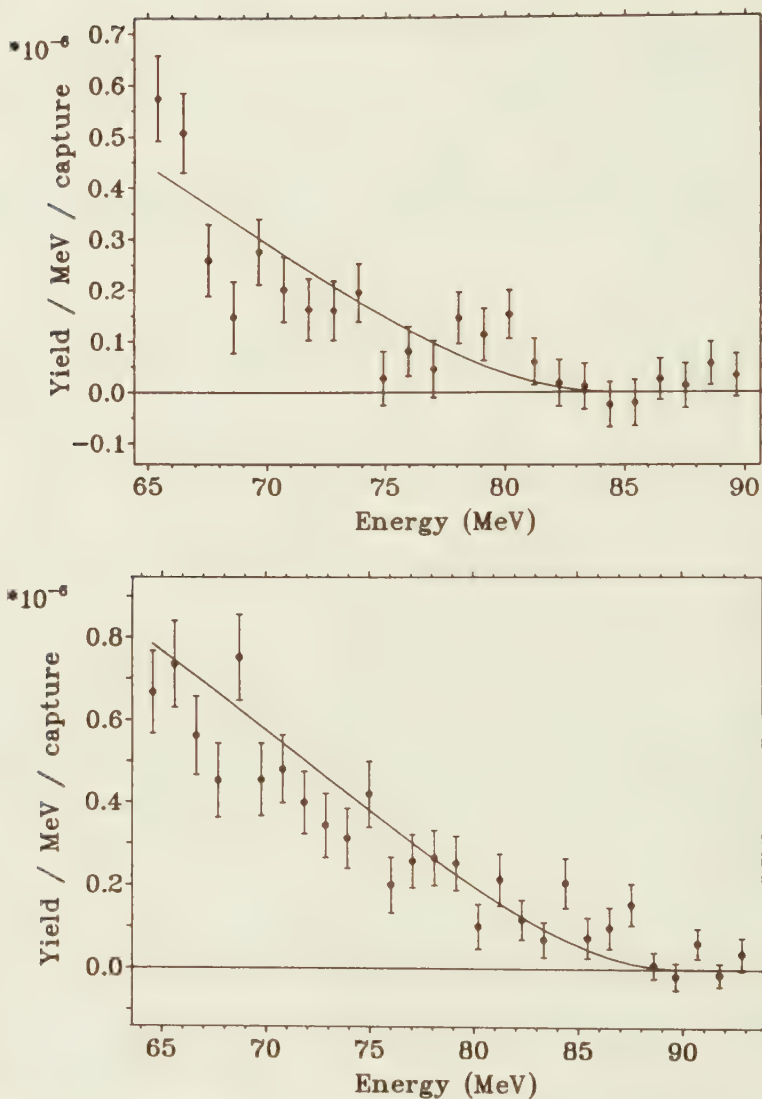


Figure 13.2: Unfolded RMC spectra for ^{200}Bi of D2 (top) and for ^{nat}Fe of D1. The solid lines are the best fits to the original data.

13.2 Absolute Normalization

The absolute Normalization is essentially given by μ_{st} and the acceptance $a(130 \text{ MeV})$. All about the μ_{st} calculation can be found in section 12.1.3.

Section 11.2 gives the details on $a_1(130 \text{ MeV})$ and section 11.6 explains the determination of $a_2(130 \text{ MeV})$.

13.3 Correction Factors

f_{cap} : The **fraction of captured muons** for an isotope is obtained from the μ -lifetime τ in this isotope as $f_{cap} = 1 - \tau/\tau_{free}$, where τ_{free} is the free muon lifetime. The experimental values for τ can be found in [ESW 66].

f_{abs} : The **target self-absorption** is calculated with a small Monte Carlo program generating a random photon distribution (of a three dimensional gaussian shape corresponding to the beam spot and the muon stop distribution) that is traced out of the target in direction of the detector. The mass absorption coefficients are taken from [HUB 80].

f_{veto} : The fraction of measuring time during which the detectors are dead by an **anticounter veto** is simply the product of the anticounter rate with the length of the veto (f_{veto} is of the order of a few per cent).

The determination of S_0 , f_n and f_{pu} , i.e. the oversubtraction factor and the fraction of neutron- and pile-up events is explained in sections 10.4, 10.2 and 10.5.

Table 13.1 lists some of the correction factors.

Target	f_{cap}	f_{abs}	f_{veto}	
			D1	D2
^{12}C	0.074 ± 0.002	0.965	-	0.98
^{16}O	0.175 ± 0.005	0.946	0.944	0.98
^{27}Al	0.606 ± 0.002	0.941	0.948	0.98
^{40}Ca	0.844 ± 0.002	0.919	0.960	0.98
^{nat}Fe	0.9064 ± 0.0006	0.908	0.952	0.98
^{185}Ho	0.9659 ± 0.0003	0.961	0.979	0.99
^{209}Bi	0.9666 ± 0.0002	0.858	0.966	0.98

Table 13.1: Correction factors

Chapter 14

Error Calculation

In the following the accuracy of all terms in the final formula for the calculation of the branching ratios 6.1, resp. 13.1 is considered.

14.1 Absolute Normalization

The absolute normalization of the photon yield is fixed by the detector acceptance and the number of muon stops in the target:

$a(130 \text{ MeV})$: The quantities entering the calculation of the absolute acceptance can be taken from equation 11.4.

The number of π_{st} was known to about 7% including the errors of $\bar{\epsilon}_{\pi_4}$ ($\pm 2\%$), of the number of accidental coincidences π_{st}^{acc} ($\pm 5.5\%$ of π_{st}), the statistical fluctuation of several stop rate measurements ($\pm 1.9\%$), and the uncertainty of the pion beam contamination ($\pm 3.4\%$).

The error of f_{abs} and f_{veto} have been neglected whereas that of the number of RPC events is purely statistical.

μ_{st} : Several measurements of the muon stop rate per μ_1 gave a fluctuation of $\sim 6\%$. The normalization of μ_{st} to the number of electron events for open intensity slits (f_{slit}) contributes about 2.5 % to the total error. Finally, the precision of the deadtime correction of the μ_1 inefficiency is about 2% which is the r.m.s. deviation of the measured μ_1 rates from the values calculated from the proton beam monitor (figure 12.3). Thus, the absolute muon stop rate can be determined to $\pm 7\%$.

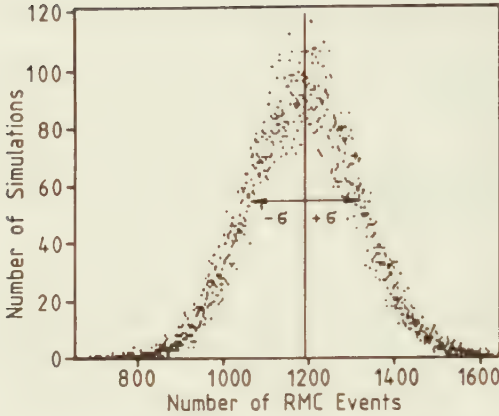


Figure 14.1: Simulated distribution of the number of events (D2) for the ^{209}Bi target.

Hence, the total error of the **absolute photon yield normalization** is **10.4 % for D1** and **11.2 % for D2**.

14.2 Number of Events

The number of events in the RMC region is essentially given by $S_O * N_{tg}(\Delta E)$. The error of this quantity is purely statistical, but rather tricky to be calculated. The difficulties are caused by the relatively complicated subtraction mechanism (section 10.6), since all the subtraction factors SF , S_{pr} , S_{cos} , and therewith the oversubtraction factor S_O are correlated. It is therefore impossible to deduce (as usual) the error of $N_{tg}(\Delta E)$ from the χ^2 -fit. Trying to express $N_{tg}(E)$ by independent quantities, a rather lengthy formula involving 21 independent variables is obtained. In order to avoid tedious derivations, a Monte Carlo method has been applied which varied the independent quantites within their normal distributions around the measured values.

Figure 14.1 shows an example of the $S_O * N_{tg}(\Delta E)$ distribution obtained by this method, and table 14.1 lists the results for all targets. As expected, the errors determined in this way are $20 \div 30$ % bigger than those estimated by means of the χ^2 -fit.

Target	Detector D1			Detector D2		
	Mean Value of $N_{tg} * S_0$	Standard Deviation	in [%]	Mean Value of $N_{tg} * S_0$	Standard Deviation	in [%]
^{12}C	-	-	-	94	63.5	67.3
^{16}O	-	-	-	401	53.5	13.3
^{27}Al	809	41.0	5.1	1145	66.2	5.8
^{40}Ca	1128	44.0	3.9	2805	86.5	3.1
^{nat}Fe	801	40.0	5.0	2320	85.4	3.7
^{165}Ho	150	22.2	14.7	230	67.3	21.4
^{209}Bi	471	49.0	10.4	1192	127	7.9

Table 14.1: Statistical errors of the number of events.

14.3 Correction Factors

f_{abs} , f_{veto} : The errors of these factors have been neglected since the corrections are rather small and there are no obvious sources of considerable uncertainties.

f_{pu} : The pile-up corrections for the heavy targets are tiny, the precision of f_{pu} thus being insignificant. In the case of ^{16}O (Detector D1), however, this factor gives the main contribution to the total error of the photon yield. Since it is too hard to estimate the uncertainty of the Monte Carlo method applied, the results of D1 for ^{16}O may not be considered as reliable.

f_n : The number of neutrons in $N_{tg}(\Delta E)$ is determined with a χ^2 -fit which simultaneously provides the error of f_n (table 10.2).

f_{cap} : See table 13.1.

Target	Detector D1		Detector D2	
	Effect of ± 0.8 MeV Shift on B_{RMC} [%]	Unsigned Mean [%]	Effect of ± 0.7 MeV shift on R_{RMC} [%]	Unsigned Mean [%]
^{12}C	-	-	+7.5 / -8.7	8.1
^{18}O	-	-	+6.4 / -6.8	6.6
^{27}Al	+7.8 / -8.2	8.0	+6.0 / -6.0	6.0
^{20}Ca	+6.6 / -6.8	6.7	+5.7 / -6.2	6.0
^{nat}Fe	+7.0 / -7.4	7.2	+6.0 / -6.2	6.1
^{185}Ho	+8.7 / -9.3	9.0	+7.4 / -8.2	7.8
^{209}Bi	+7.4 / -7.5	7.5	+7.9 / -8.2	8.1

Table 14.2: Effect of energy calibration on the error of the branching ratios.

14.4 Energy Limits and Resolution

The main part of the RMC spectrum is concentrated at low energies. An interval of 1 MeV at the cut-off energy (64 MeV) contains about 10% of the total observable spectrum. The **precision of the energy calibration** therefore plays an important role. The energy $\leftrightarrow$ channel relation (at ~ 60 MeV) is known to ± 0.8 MeV for D1 and to 0.7 MeV for D2. The effect of this on the error of the branching ratios is presented in table 14.2.

The influence of the **shape of the resolution function** $R(E, E_0)$ on the value of B_{RMC} is fairly weak. A change of its width by 30% has only a 2% effect on the result of the photon yield.

14.5 Theoretical Uncertainties

As long as the photon yield has to be extrapolated below the experimentally accessible energy region, the result strongly depends on the applied theoretical spectral shape. This can be illustrated by means of the poly-

nomial 'PP'. If, for example, the accessible energy region goes from 64 to 90 MeV and the photon spectrum is extrapolated down to 57 MeV, the estimated total yield increases by 3% if E_0 is changed from 90 MeV to 89 MeV. For an E_0 of 80 MeV a change of 1 MeV has already an effect of more than 4% on the total yield above 57 MeV.

As table 7.2 proves, E_0 is rather uncertain (by several MeV) for some spectra.

In order to provide information on the photon yield which is fairly independent of any theory, the branching ratios for photon events >64 MeV are given in the appendix (table B.2).

Part III

APPENDIX

Appendix A

Compilation of Spectra

In the following the fitted RMC spectra for all targets are shown. As figure 7.3 illustrates it is of no use to display the best fits for several theories since they are nearly indistinguishable.

A.1 Detector D1

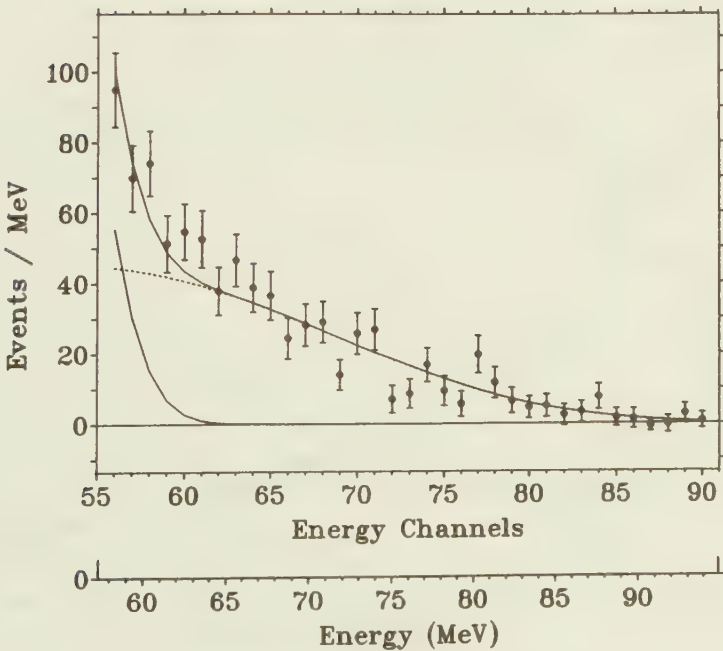


Figure A.1: ^{16}O spectrum fitted with CG

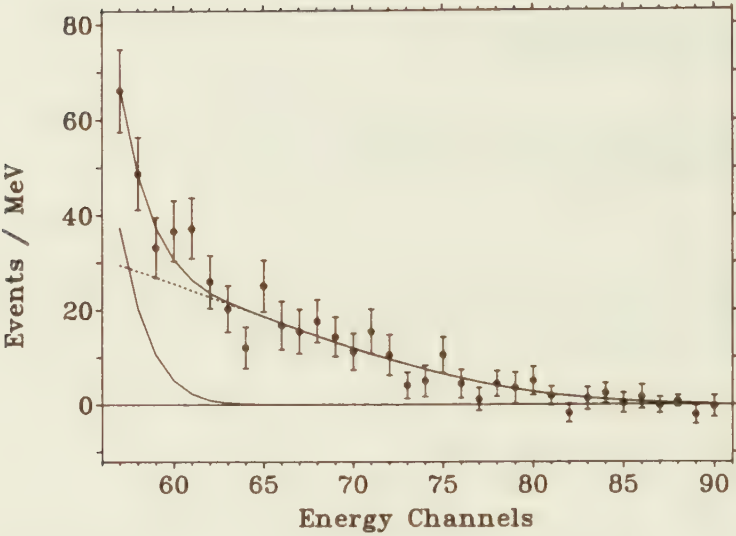


Figure A.2: ^{27}Al spectrum fitted with PP

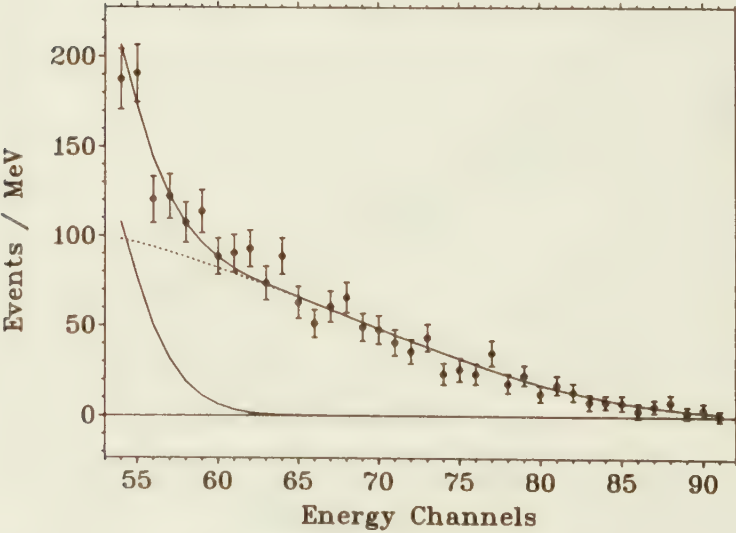


Figure A.3: ^{40}Ca spectrum fitted with CH

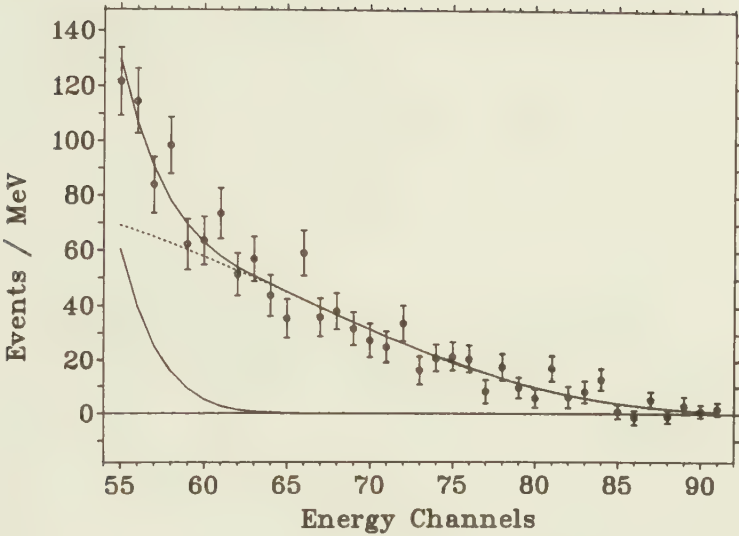


Figure A.4: ^{nat}Fe spectrum fitted with PP

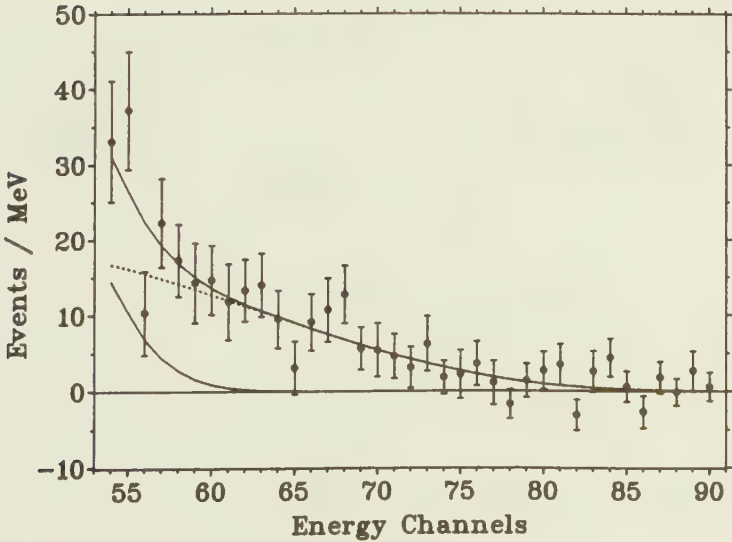


Figure A.5: ^{165}Ho spectrum fitted with PP

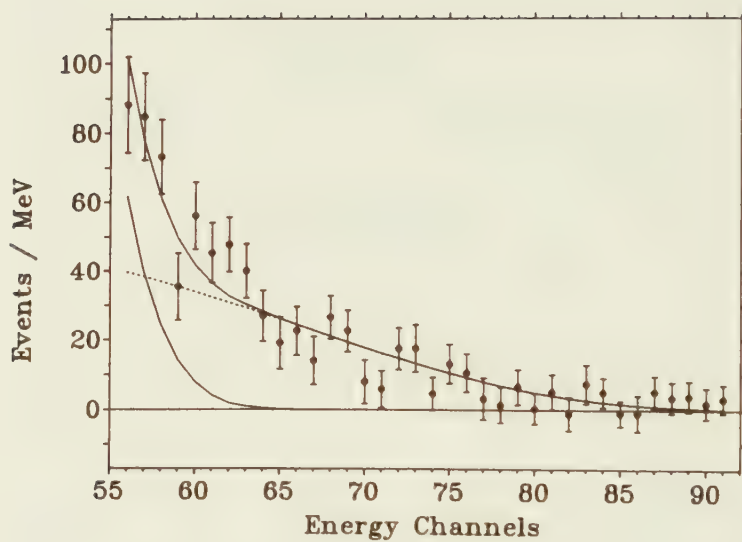


Figure A.6: ^{209}Bi spectrum fitted with PP

A.2 Detector D2

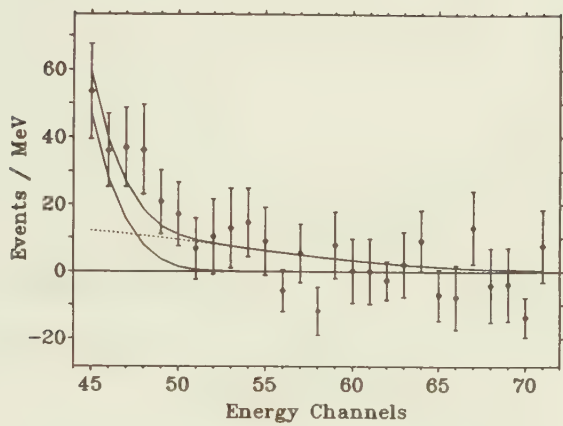


Figure A.7: ^{12}C spectrum fitted with CG

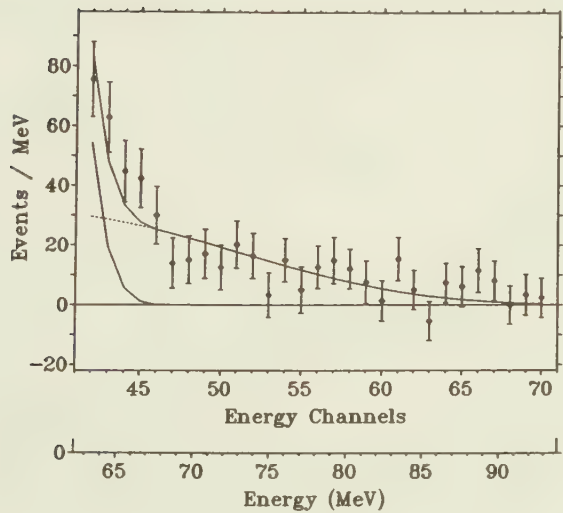
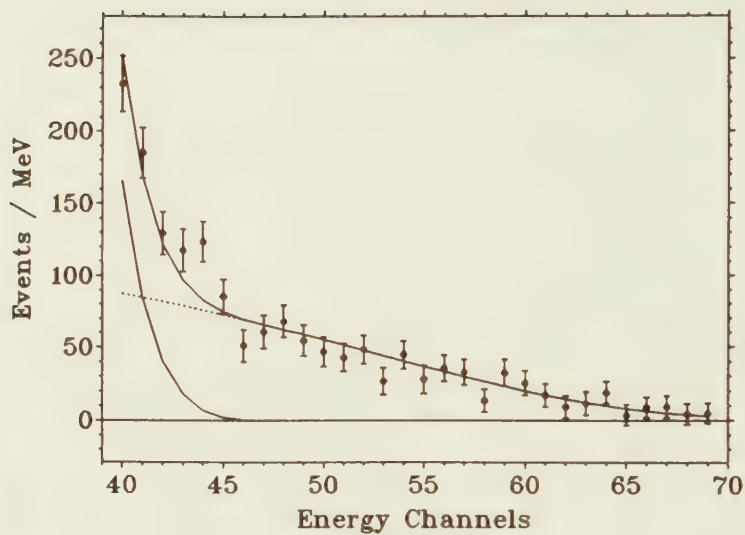
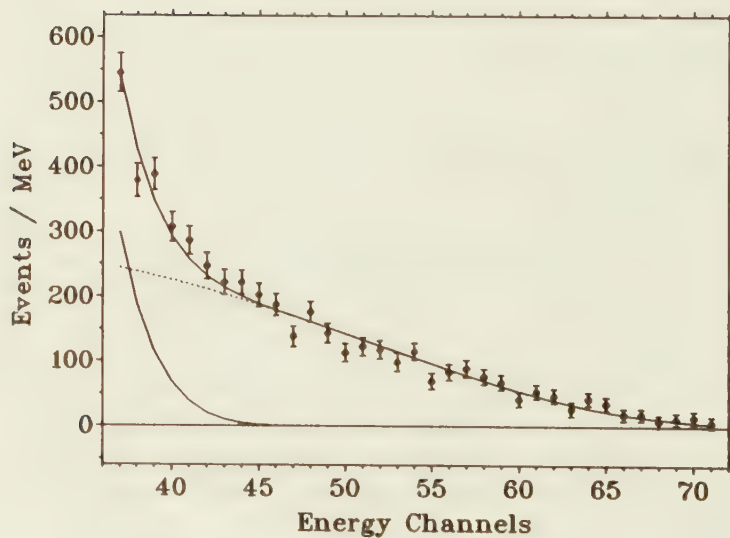


Figure A.8: ^{16}O spectrum fitted with CG

Figure A.9: ^{27}Al spectrum fitted with PPFigure A.10: ^{40}Ca spectrum fitted with CH

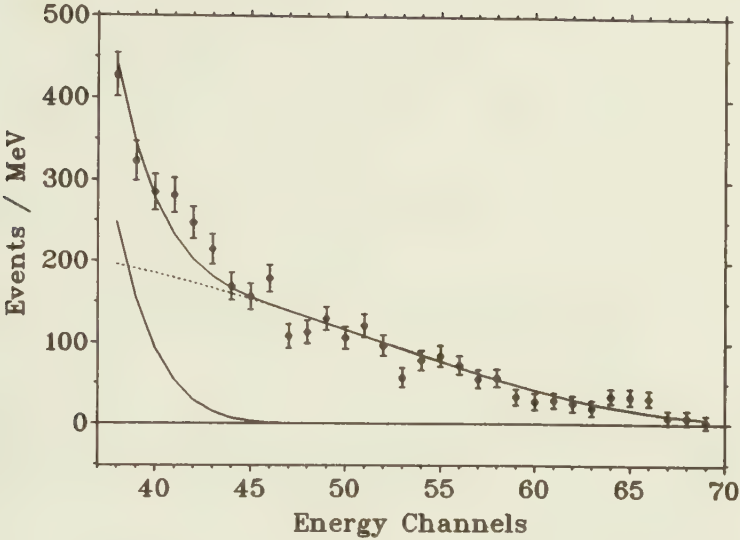


Figure A.11: ^{nat}Fe spectrum fitted with PP

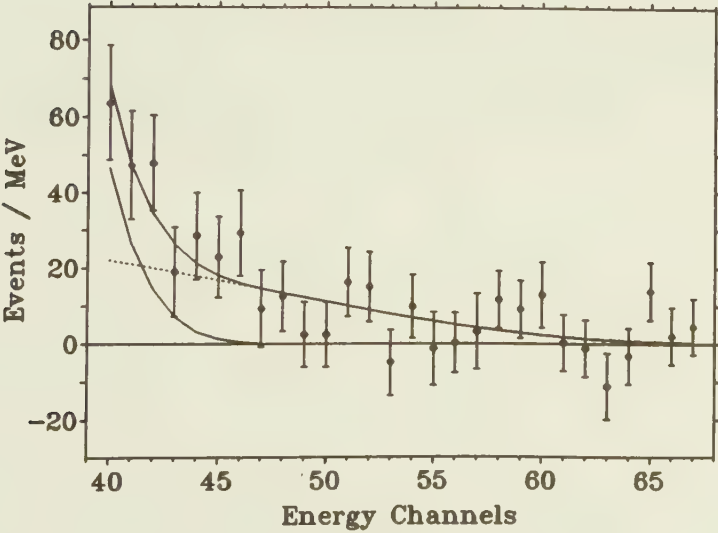


Figure A.12: ^{165}Ho spectrum fitted with PP

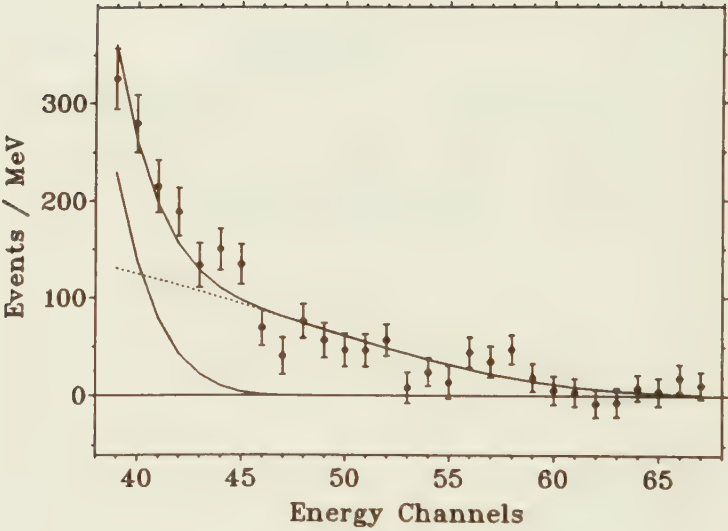


Figure A.13: ^{209}Bi spectrum fitted with PP

Appendix B

Varia

B.1 Run Statistics

In table B.1 some information on the run statistics is given. The uncertainty of the stop rate and the dimensions of the Al target originates from a confusion of two existing Al samples.

B.2 Photon Yield above 64 MeV

Table B.2 lists the RMC branching ratios for the energy region >64 MeV. They are expected to be rather independent of the theoretical spectrum shape used for the χ^2 -fit because no extrapolation to experimentally not accessible energies is made.

Target	Total Number of μ_1 -counts [10^9]	Mean μ_1 -rate [MHz]	Fraction Stopped in Target	Target Thickness [g/cm^2]	Number of Photon events [10^3]	
					D1	D2
^{12}C	270.	20.4	.15	5.1	-	28.7
^{16}O	430.	23.9	.19	5.0	54.0	317.
^{27}Al (D1)	700.	27.9	.18/.15	5.4/3.8	54.5	-
^{27}Al (D2)	507.	28.0	.18/.15	5.4/3.8	-	253.
^{40}Ca	696.	21.7	.18	5.0	48.2	259.
^{nat}Fe	701.	27.5	.18	5.6	27.2	186.
^{165}Ho	575.	19.7	.07	1.9	10.0	85.4
^{209}Bi	1422.	27.8	.11	4.5	30.4	237.
Empty	1133.	26.7	-	-	6.7	104.

Table B.1: Additional statistics. μ_1 counts are deadtime corrected.

Target	Theory	Detector D1		Detector D2	
		Branching Ratio [10^{-5}]	E_0 [MeV]	Branching Ratio [10^{-5}]	E_0 [MeV]
^{12}C	CG	-	-	1.6 ± 1.0	-
^{16}O	CG	1.81	-	1.38 ± 0.27	-
	PP	-	-	1.39 ± 0.27	89.9
	SL	-	-	1.37 ± 0.26	85.5
^{27}Al	PP	$0.99^{+0.31}_{-0.13}$	86.8	$1.06^{+0.34}_{-0.15}$	90.8
	SL	$1.00^{+0.31}_{-0.13}$	83.1	$1.06^{+0.34}_{-0.15}$	85.8
^{40}Ca	CH	1.34 ± 0.18	-	1.39 ± 0.18	-
	PP	1.29 ± 0.17	92.1	1.40 ± 0.18	92.6
	SL	1.34 ± 0.18	86.5	1.39 ± 0.18	87.9
^{nat}Fe	PP	0.92 ± 0.13	89.7	1.10 ± 0.15	90.6
	SL	0.92 ± 0.13	85.4	1.11 ± 0.15	86.3
^{165}Ho	PP	0.44 ± 0.09	83.7	0.30 ± 0.10	85.1
	SL	0.43 ± 0.09	78.1	0.28 ± 0.09	73.5
^{209}Bi	PP	0.32 ± 0.06	88.5	0.34 ± 0.06	84.2
	SL	0.32 ± 0.06	81.2	0.34 ± 0.06	77.6

Table B.2: RMC branching ratios for photons with energies >64 Mev.

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I would like to conclude this thesis by the last words that God addressed to his creation:

We apologize for the inconvenience

I apologize to Douglas Adams for copying this message from his book 'So long and thanks for all the fish'.

